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PREFACE

In 1885 a small group of Hungarian mathematicians and physicists started holding regular meetings at a restaurant near the University in Budapest. Here they heard the latest news in science and learned about each other's research, thereby encouraging scientific activity in Hungary.

In 1890, *Loránd (Roland) Eötvös*, professor of physics, mailed an invitation to his lecture on Terrestrial Gravitation and its Measurement. His message referred to further goals: "Dear Gentlemen! We hope that by meeting here we are about to take the first step towards assembling again and again with a similar purpose and get into even closer contact. With full respect, yours sincerely, Baron Roland Eötvös."

In 1891, the *Mathematical and Physical Society* was established. Referring to the society, Eötvös wrote: The goal of our meetings is "the further development of science by the spoken word, and the publication in a journal of all results worth the attention of the experts. This does not seem to be more than the goal of the study groups of the students. Still, in this case, we give it the attention it deserves, and our work will have merit for fulfilling an important need. If we succeed in making every Hungarian teacher of mathematics and physics a real mathematician and physicist, then we not only serve the schools, but we raise the level of science in our country. Carrying out the task of self-education with dedication and seriousness has the further result that, in the future, new researchers and scientists arise out of our group. I herby declare the Mathematical and Physical Society founded."

The first president of the new society was Loránd (Roland) Eötvös, rector of the Budapest University of Sciences. The vice-presidents were *Gyula König*, the Rector of the Technical University (an excellent developer of mathematical analysis) and *Ágoston Schmidt* (a secondary school teacher). The initial members numbered 300. (Now both the János Bolyai Mathematical Society and the Roland Eötvös Physical Society consider themselves offspring of this society.)

The year 1894 was important in the history of Hungarian science and education. A school teacher named *Dániel Arany*, started editing the *Középiskolai Matematikai Lapok*, a mathematical problem solving journal for secondary school students. This was the second journal of its type in Europe preceeded only by Vuibert's *Journal de mathématiques élémentaires*, started in France in 1876. Each issue of the Középiskolai Matematikai Lapok contained a number of selected excercices from mathematics and shortly thereafter from physics, as well as solutions to past months' problems and a list of those pupils who sent in correct solutions. Now the *Középiskolai Matematikai és Fizikai Lapok* is published by the János Bolyai Mathematical Society and the Roland Eötvös Physical Society in 7000 copies per month. It regularly prints the best solutions of the students and the names of the best problems solvers. One can find the names of famous scientists like Lipót Fejér, Tódor (Theodore von) Kármán, George Pólya, Frigyes Riesz, Marcell Riesz, Gábor Szegő, Pál (Paul) Erdős, on the list of successful students.

In 1894, the new society decided to organize a mathematical competition for students just finishing high-school. In 1916, the mathematical competition was augmented by a physics competition. With the exception of a few small gaps during the world wars, the competition has been held every year. Since then for one hundred years these competitions enjoy great respect. They bring fame (and university admission) to the winning students, and fame to their teachers and schools as well.

These competitions were the place where many (later) famous mathematicians and physicists had their first success; in mathematics, they included L. Fejér (1897), T. Kármán (1898), M. Riesz (1904), G. Szegő (1912), E. Teller (1925), and László Tisza (1915); in physics, Leó Szilárd (1916) and E. Teller (1925). Among the winners of the *József Kürschák Mathematics Student Competitions* and the *Roland Eötvös Physics Student Competitions* one can find the names of internationally renowned mathematicians and physicists even nowadays.

Both the competitions and the journal have, during the one hundred years of their history, played an important role in discovering talented boys and girls. Anyone analysing the source of success of mathematicians and physicists of Hungarian origin in our century is compelled to emphasize the role of these two organizations.

The end of the 19th century saw rapid development of mathematics and physics worldwide as well as in Hungary. Loránd (Roland) Eötvös organized the first *Teacher's Seminar* in 1895 for the training of physics teachers from all over the country. Since then, the close contact between active researchers, university professors, and school teachers has been a significant goal for both societies. (Professor Rudolf Ortvay introduced the most active school teachers in this way to quantum mechanics already in 1930.)

Since those years, the stormy 20th century has passed by, but Eötvös's dream was realized: the two societies have served education, research, and progress without interruption. This centennial offers us an opportunity to introduce our societies to you.

Ákos Császár

Honorary President of the
János Bolyai Mathematical Society

George Marx

Honorary President of the
Roland Eötvös Physical Society

János Bolyai Mathematical Society

The aim of the Bolyai Society is to help and coordinate the activities of mathematics teachers, pure mathematicians and applied mathematicians. Work is accomplished by three corresponding sections within the society.

The function of the *Education Section* is twofold. On one hand it helps public education by publishing material, organizing lectures and discussions, and supporting educational research. Recently, a computer network was started for such purposes. The highlight is the annual conference of mathematics teachers. The section's other major aim is to discover and support mathematical talent.

The *Section of Pure Mathematics* and the *Section of Applied Mathematics* organize lectures and international conferences and publish their proceedings. For instance, the society will organize the 2nd European Congress of Mathematics, in Budapest in 1996. Naturally, the three departments do not work in isolation: interaction is a very important factor in their effectiveness.

Some people claim that Hungarian mathematics is very strong. Others disagree with this statement. In any case, even if only partially true, there can be no argument with the fact that the search for and support of mathematical talent has a central role in the activity of our society. We organize an annual nationwide mathematical contest for six different age groups between 12 and 18 (the population of Hungary is only 10 million!); the best students are then prepared for the International Mathematical Olympiad. There is also an interesting contest for undergraduate students: contestants are given ten days to solve problems requiring a broad knowledge of different areas of mathematics. The society also gives prizes for the best research produced by students for their master's theses (the Kató Rényi Prize). Young mathematicians can receive other prizes for outstanding research in pure and applied mathematics (the Géza Grünwald Prize and the Gyula Farkas Prize, respectively). Finally, there is even a prize for the older generation: the Tibor Szele Prize, awarded for training outstanding researchers.

But one of the most important parts of this search for talent is KöMaL (KöMaL is the popular abbreviation of Középiskolai Matematikai és Fizikai Lapok, the high-school mathematics and physics journal). This journal introduces young students to non-trivial mathematical problems and teaches them to work hard and steadily, preparing them for competitions.

It is a miracle that this journal has not ceased to exist in spite of the several political changes, wars, and revolutions in Hungary during the last 100 years. An important reason is that many of the scientific, economic and even political leaders were participants in KöMaL, sending solutions regularly. Many prominent people understand the importance of this activity. I feel that the Lord must have been a solver, too. This gives me hope that there will be financial support for the next 100 years as well.

Gyula Katona
Secretary General of the János Bolyai Mathematical Society

Roland Eötvös Physical Society

Physics problems have appeared in the KöMaL since its genesis 100 years ago, and later, with the help of the Eötvös Society, the *Physics Column* became a permanent addition. For this reason, the Eötvös Society also celebrates this centennial.

The society has 1305 members, 18 specialty sections, and 20 regional branches.

The Eötvös Society launched the monthly *Fizikai Szemle* in 1950 with the optimistic words of *Károly Novobátzky*: "Physics develops with enormous speed, this is the most important science of the world, its importance is unsurpassable by anything else. It was not without reason that Einstein called physics the fifth superpower. Experimental physics reaches for the innermost secrets of matter with extremely sophisticated and expensive methods, while theoretical physics explores the possibilities of logically structuring natural phenomena. In Hungary, the impatient desire to catch up with what we have missed for centuries turns into action in physics as well."

The Eötvös Society extended its reach to the whole country, organizing local branches nationally since 1951 and has constantly changed the locations of its biannual meetings. It has become perhaps the most important organizer of international conferences in Eastern Europe since the 1960's. (It hosted several conferences of the European Physical Society, International Union for Pure and Applied Physics, International Atomic Energy Agency, International Society of Astronautics, and UNESCO.) On a lake shore street in Balatonfüred, linden trees planted by *Paul A. M. Dirac*, *Richard Feynman*, *Miklós (Nicholas) Kürti*, *Benoit Mandelbrot*, *Sir Nevil Mott*, *Rudolf Mössbauer*, *Bruno Pontecorvo*, *Jurij V. Prohorov*, *Ede (Edward) Teller*, and *Jenő (Eugene) Wigner* preserve the memories of their visits and lectures at Balaton conferences.

The annual conferences of physics teachers attract several hundred participants from all over the country, they play a central role in in-service-training and the exchange of methods. The Eötvös Society continues to support and organize physics competitions and conferences. Although the Eötvös Competition is perhaps the most famous, it is not the only one run by the Eötvös Society. It would take a long list to enumerate them all, let us stop by saying that the long list begins in primary school. Competitions carry the names of famous teachers, such as *István Hatvani*, *Ireneus Károly*, *Sándor Mikola*, *József Öveges*, *Miklós Vermes* etc. Each year over 10,000 students participate in these competitions.

Dezső Kiss
President of the Roland Eötvös Physical Society

Chapters from the History of Mathematics and Physics in Hungary

The Round Table of Mathematicians

The history of the Mathematical Journal for Secondary Schools goes back more than one hundred years. The foundation of the journal had close connections with the boom in scientific life at the turn of the century and the foundation of the Mathematical and Physical Society. Let's go back to the previous century, when two of the most important centers of Hungarian culture were the Technical University in Budapest and the Bolyai University in Kolozsvár, Transylvania. The Round Table of Mathematicians was formed in the Autumn of 1885, in Budapest. There was no chairman and no ground rules. The Founding Fathers were *Loránd (Roland) Eötvös*, *Jenő Hunyadi*, *Ágoston Schmidt*, *Kálmán Szily*, *Gyula König*, *Gusztáv Rados*, and *Manó Beke*; the members were professors of mathematics and physics at the Technical University and teachers from various secondary schools. They themselves gave most of the talks. Mathematics and physics were not separated as sharply as they are today. Several mathematicians liked to work also on physics, and many professors found it hard to distinguish their respective fields. In those days many scientists tried to become polyhistors. As Manó Beke wrote in 1911: *"We mathematics teachers enjoyed these evenings very much; we looked forward to them. We profited a lot, and received much material for further study. It was not the main purpose to present new results, although we heard new results from Hunyadi or Szily before they presented them at the Academy, and König presented a paper about the same time to us as to the Academy. Our goal was to get acquainted with the most important and most interesting recent publications. Just as we enjoyed the scientific lectures, the 'mathematical dinners' following them were also memorable. Here the best researchers mixed with the teachers of the secondary schools."*

It was the merit of Eötvös to recognize the importance of an active scientific community to the cultural development of a nation. It was Eötvös himself, who in 1890, came up with the idea of an organized society that would bring scientists together. Instead of using empty words, he had the following motto: "Let us learn from each other in order to teach better."

Gyula König formulated the condition of membership as *"not just anybody, but everybody who works on mathematics and physics and their popularization can be nominated."*

The ground rules based on these ideas were accepted by the Ministry of the Interior on August 21, 1891, and the newborn society started recruiting members.

The Mathematical and Physical Society

On November 5, 1891, Loránd Eötvös was elected president of the society. One of the most important goals of the society was to start and secure the success of a mathematics and physics journal. Similar journals, such as the "Technical University Notices" (1876–1878), existed earlier, but for financial reasons lasted only two years. The Academy of Sciences published its "Mathematical and Natural Sciences Bulletin" with the special goal of publishing only original results.

The first issue of the *Matematikai és Fizikai Lapok* (*Mathematics and Physics Journal*) appeared in 1891. Eötvös summarized its goal as follows: "...to popularize science and not to publish independent results, as other journals successfully achieve this goal. We publish popular articles about recent results for experts to use them in their research..." In addition to the scientific articles, the journal also published articles to help in teaching mathematics and physics.

Among the authors of the articles were many world famous mathematicians. The articles included *Felix Klein's* "The Erlangen Program," the first Hungarian version of *János Bolyai's* Appendix (originally written in Latin), *Friedrich Gauss's* work on surfaces, *Lipót Fejér's* famous doctoral thesis, and many important results by *Frigyes Riesz*, *Zoárd Geőcze*, *Alfréd Haar*. It is impossible to list all the famous mathematicians who published their results and thoughts in these pages.

Rudolf Ortvay *Lipót Fejér*

The first editors of the journal were *Gusztáv Rados*, *Géza Bartoniek*, *Lipót Fejér*, *Győző Zemplén*, *Rudolf Ortvay*, *Dénes König*. From 1930 on, the journal was helped by young graduates, who published their theses without honorarium. The publication of the journal was not without problems, mostly financial, and often issues would be skipped, or two issues would be published together in one joint issue. The journal has managed to survive all its problems and its offsprings, *Matematikai Lapok* and *Fizikai Szemle* still exist. The main goals of the journal remain unchanged: *Matematikai Lapok* publishes informative and original results, a problems section, solutions to previous problems, book reviews and events of the society. The journal *Fizikai Szemle* (*Physics Review*) began in 1950 and is published today by the Roland Eötvös Physical Society in 2500 copies. This monthly is similar to "Physics World", "Physikalische Blatter", or "Physics Today", but has a certain international character, as it has published and continues to publish papers by Hungarian born scientists living overseas like *Zoltán Bay*, *Miklós (Nicolas) Kürti*, *János (John von) Neumann*, *Ede (Edward) Teller*, and *Jenő (Eugene) Wigner*.

The Year 1894

Alfréd Haar *Frigyes Riesz*

Loránd (Roland) Eötvös (1848–1919) was the founder of professional research in physics in Hungary. His torsion pendulum played an important role in oil prospecting both in Europe and America. He obtained his doctor's degree (together with the mathematician *Gyula König*) in Heidelberg in 1870. He became professor at the age of 30 at the University of Budapest (later to be named after him in 1950). He was the founder and first president of the Hungarian Mathematical and Physical Society. He remained its president until his death. He obtained world fame through his experiments verifying the proportionality of inertial and gravitational mass up to 8 decimals and thus preparing the path to General Relativity.

One of the high points of his career came in 1894 when he was nominated to be the Minister of Religion and Public Education. Members of the Mathematical and Physical Society commemorated his nomination by launching the *Eötvös Competition* for talented students finishing secondary school.

The year 1894, a hundred years ago, marked three important events:

— The *Eötvös Competition* was started.

— *Eötvös College* was established for those university students who wished to become teachers. The college was named after *József (Joseph) Eötvös*, Loránd's father, a liberal novelist and former Minister of Education and Culture who devoted his life to the cultural development of Hungary.

— The *Középiskolai Matematikai Lapok* ("Mathematical Journal for Secondary Schools") was established. The publication was targeted at secondary school students, offering papers and problems in mathematics and later expanded to include articles on physics.

Mathematical Journal for Secondary Schools

In November of 1893, *Dániel Arany*, a high school teacher from the city of Győr, decided to found a mathematical journal for high school students. The journal, the Középiskolai Matematikai Lapok (Mathematical

Dániel Arany *László Rátz* *Sándor Mikola*

Journal for Secondary Schools), published its first edition in December 1893, displaying its goals on the cover: "to give a wealth of examples to students and teachers." The journal's first edition appeared on January 1, 1894, and from that time on ten issues a year were published. We know of 132 subscribers in the first year. In the first three years, a total of 239 problems were published, for which 1055 solutions arrived from 151 students, representing all parts of Hungary. The journal was successful, but struggled with financial problems.

In 1896, the editorship was taken over by *László Rátz*, a teacher at the Lutheran Secondary School in Budapest who had previously contributed to the journal. Dániel Arany continued to work enthusiastically on the journal, and in 1907, they were joined by *Márk Antal*, a business school teacher. In 1897, *Sándor Mikola* got his first job at the Lutheran Secondary School. He eagerly offered to write articles on physics for the journal, a suggestion that was well received by László Rátz, his colleague. In the tenth year of publication, a physics problems column was added to the journal. To the yearly 120–180 problems, 2500–3000 solutions were received, and among the authors of the solutions were such names as Lipót Fejér, Mihály Fekete, Alfréd Haar, Tódor Kármán, Dénes König, György Pólya, Tibor Radó, Frigyes Riesz, Marcell Riesz, Gábor Szegő, Győző Zemplén...

László Rátz was forced to leave the journal at the start of the First World War. With his departure, the first successful period of the journal ended and publication was interrupted for more than a decade.

Based on articles written by
Elizabeth Fried, George Marx, Gyula Radnai, János Surányi, and **Barna Szénássy.**

The Eötvös Competition

The Idea of the Competition

Géza Bartoniek *Gusztáv Rados*

One hundred years ago the decisive act in starting of the Eötvös Competition was due to Loránd (Roland) Eötvös, appointed as Minister of Culture and Education. This doesn't mean however that Eötvös was the initiator of this competition. On the contrary, the idea of this competition came from the other leaders of the Mathematical and Physical Society. They launched this competition on the occasion of the appointment, in honour of the President. It was prepared as a pleasant surprise for him!

Who were these people? The vice-president: *Gyula König* (1849–1913), mathematician and the secretary of the society; *Géza Bartoniek* (1854–1930), physicist and later the director of Eötvös College; and *Gusztáv Rados* (1862–1942), mathematician and later the main librarian at the Technical University. They suggested that the competition should have a first and a second prize and these should be designated as the first and second Eötvös Prizes. Furthermore, the prizes should be accompanied by rewards of 100 and 50 gold crowns from the budget of the society. The proposition became a resolution and since then the organization and layout of the competition is one of the important activities of the society.

The prizes and rewards have always been given by the president of the society in the form of a ceremonial act. While giving his congratulations at these occasions, Eötvös never forgot to mention the merits of the teachers of the prizewinners.

The competition had been named after Eötvös from the beginning, although officially its name was "Student Competition". Only after the death of Roland Eötvös, when the society took his name, did the competition officially become the Eötvös Competition.

Before launching the Eötvös competition — a meeting in the Ministry.

In 1891, three years before the Eötvös competition was started, a small conference was organized in the Ministry of Culture and Education. Participants were invited and the topic for discussion was the possibility of a student competition in the secondary schools.

The Minister raised the following question for the participants: "Would it be possible in our country, as in France, to use open competitions to stir the enthusiasm of our youth for studies?

The proposed answer to be discussed was given by the head of the secondary school department of the ministry. He suggested to have competitions in language, literature, history, and mathematics.

The contests would be similar to the final exams in secondary schools, but not intended for all students, only for the bests. To avoid disturbing the normal school-life, these competitions were planned to be taken after the final exam and before the start of university studies. The prizes for the winners would be scholarships for further studies.

One third of the jury would come from secondary schools, two thirds would be independent cultural authorities.

This official proposal was well received. Only one participant debated the suggestion; Eötvös, the new rector of the University. According to the proceedings: "Baron Eötvös doubts the benefits of the competitions. He does not consent to using university scholarships for this aim as there are very few and present distribution has proved to be good."

It is not easy to guess the motivation which led Eötvös to speak against the proposed competitions. Was he afraid that the University would lose the right to distribute the scholarships? Did he want to hinder the revival of the Jesuit tradition of student competitions? Or did he not trust the independence of the authorities?

Knowing Eötvös's opinion, the birth of the Eötvös Competition is even more interesting. However, an important fact should be mentioned: Gyula König didn't take part at the meeting in the ministry.

The birth of the Eötvös competition

After this conference, the idea of student competition in all secondary school subjects was abandoned for decades.

Three years later Eötvös became the Minister of Culture and Education. In his absence the Mathematical and Physical Society "... accepts the following decision in order to commemorate the appointment of the president: To promote success in teaching and learning mathematics and physics, the society hereby decides to organize every autumn an open competition in mathematics and physics for students who have their final exam in that year." The competition is to be held both in Budapest and Kolozsvár at the same time. The purpose of the competition is to offer an occasion for the candidates to prove their talent and gift in cultivating the sciences mentioned." It should be noted that the statute speaks of talent and gift in cultivating mathematics and physics, not of the knowledge of the subject! In this aspect, the Eötvös Competition surpassed the traditional competitions known to that date.

It is possible that the formal idea of this competition was based upon the proposition of the Ministry, but the actual differences are very important:

The competition is open for all students, not just the best ones. Any auxiliary material may be used — not only logarithmic tables.

This is partly why the Eötvös Competitions have been able to retrain their predicted value.

The point is that this competition was launched not by a bureaucratic ministry wanting to regulate everything, but by a merry, liberal company gather around a table in a restaurant, later to become known as the Hungarian Mathematical and Physical Society.

The idea of this competition was a victory for Gyula König, a "king" of mathematics at that time. It seems clear that the leaders of the society succeeded in finding a good aim which served the approval of Eötvös, a devoted physicist of a liberal mentality.

Bolyai Mathematical Society — Eötvös Physical Society
Kürschák Competition — Eötvös Competition

Originally the Eötvös Competition was a competition in mathematics.

In 1916 Ireneus Károly (1854–1929), later vice president of the society, donated a 2000 gold crown endowment and initiated the competition also in physics in the framework of the society. Károly was a professor of physics in Nagyvárad (Oradea), a catholic monk and a former student of Eötvös.

Between the two World Wars, the mathematical competition of the society was called the Eötvös Competition while the competition in physics was called the Károly Competition. After the Second World War the society divided into two parts. Since Eötvös was an eminent physicist, the physical society took his name. The mathematical society chose the name of *János Bolyai* (1802–1860), an eminent Hungarian mathematician.

János Bolyai was an army officer who, at a far-eastern garrison, created non-Euclidean geometry in 1825. The 21-year old lieutenant wrote to his father, the mathematician Farkas Bolyai: — "Out of nothing I have created a new, different world." — He speculated that the geometry used in nature may differ from that of Euclid and may be interrelated with gravity. The fact that the Hungarian Mathematical Society commemorates János Bolyai and has propagated his memory all over the world for almost forty years can be considered a historical rehabilitation of this great mathematical genius of the last century. The János Bolyai Mathematical Society was established in 1947, the Roland Eötvös Physical Society in 1949. This was followed by the survival of the student competitions. The physical competition was named after Loránd (Roland) Eötvös and the mathematical competition after József (Joseph) Kürschák.

The mathematician *József Kürschák* (1864–1932) has all the merits to be commemorated by a student competition in mathematics. He was involved in running the competition from its beginning, in proposing problems, and in collecting interesting solutions. He edited this collection in a book, known as the Hungarian Problem Book. In actively popularizing the competition, he served the interest of mathematics as well as that of the gifted students. Today the Kürschák and Eötvös competitions are the followers of the old, traditional Eötvös and Károly competitions; followers, concerning both the aims and the effectiveness of finding mathematical and physical talent.

A big race or a good effort?

When we observe that the name Eötvös always appears in the name of the student competitions, we may ask ourselves: Is it really due to him? The naïve observer may think that Eötvös was a fanatic of competitions and/or a fountain of ideas for tricky problems. This observer would be wrong. Eötvös never gave himself to contests or competition with others. He did not like competing but he did like effort. He always fought with himself, with the problems he gave himself. He considered the problem a challenge of nature and not a puzzle given by some person. Perhaps this is the true motivation for associating Eötvös with these competitions.

The Eötvös Competition is indeed a good effort, not a big race. Eötvös's spirit is the guarantee that the problems posed are of a high standard, and challenge the gifted spirit of the pupils rather than their knowledge. It may be instructive to recall the moments of the birth of the competition in these days, of a changing Central Europe. Let's consider the Eötvös Competition as a successful pedagogical experiment, the outcome of which we already know. The scientific careers of the winners have demonstrated the success of the concepts: 1. Not the "race", but the effort; 2. Not the knowledge but the know-how!

Gyula Radnai

First and Second Prize Winners
of the Eötvös Mathematics Competitions (1894–1943)

1894: 1. *Seidner, Mihály*; 2. *Pap, Pál.* **1895:** 1. *Riesz, Emil*; 2. *Pilczer, Ignác.* **1896:** 1. *Visnya, Aladár*; 2. *Zemplén, Győző.* **1897:** 1. *Fazekas, Bernát*; 2. *Fejér, Lipót.* **1898:** 1. *Kármán, Tódor Tivadar*; 2. *Gróffits,* *Gábor.* **1899:** 1. *Kornis, Ödön*; 2. *Spiczer, Ödön.* **1900:** 1. *Juvantz, Iréneusz*; 2. *Szmodics, Kázmér.* **1901:** 1. *Póka, Gyula*; 2. *Baranyó, Ernő.* **1902:** 1. *König, Dénes*; 2. *Szmodics, Hildegárd.* **1903:** 1. *Haar, Alfréd*; 2. *Horvay (Kronstein), Béla.* **1904:** 1. *Riesz, Marcell*; 2. *Fuchs, István.* **1905:** 1. *Ujj, Gyula*; 2. *Neubauer, Constantin.* **1906:** 1. *Erdős, Vilmos*; 2. *Gotléb, István.* **1907:** 1. *Tolnai, Jenő*; 2. *Domokos, György.* **1908:** 1. *Orphanides, Etelka*; 2. *Kudlák, Lajos.* **1909:** 1. *Raj, László*; 2. *Lukács, Ferenc.* **1910:** 1. —; 2. *Sebestyén, Ödön.* **1911:** 1. *Hlucsil, Károly*; 2. *Klein, Gábor.* **1912:** 1. *Szegő, Gábor*; 2. *Neményi, Pál.* **1913:** 1. *Radó, Tibor*; 2. *Filep, Lajos.* **1914:** 1. *Zigány, Ferenc*; 2. *Prónai, Jenő.* **1915:** 1. *Boskowitz, Alfréd*; 2. *Krbek, Ferenc.* **1916:** 1. *Koródi (Kornfeld), Albert*; 2. *Hajnal, Kálmán.* **1917:** 1. *Pomázi, Z. Celesztin*; 2. *Sárospataky, Lajos.* **1918:** 1. *Reuss, Endre*; 2. *Rédei, László.* **1922:** 1. *Kalmár, László*; 2. *Schmidt, Vilmos.* **1923:** 1. *Ländler, György*; 2. *Izsák, Miklós.* **1924:** 1. *Körner, Endre.* **1925:** *Fuchs, Rudolf*; *Teller, Ede*; *Tisza, László.* **1926:** 1. *Bakos, Tibor*; 2. *Winkler, József.* **1927:** 1. *Beke, Gyula*; 2. *Ság, Miklós.* **1928:** 1. *Svéd (Schossberger), György*; 2. *Schlüsser, Endre.* **1929:** 1. *Párducz, Nándor*; 2. *Szolovics, Dezső.* **1930:** 1. *Gallai (Grünwald), Tibor*; 2. *Déman, Pál.* **1931:** 1. *Sebők, György*; 2. *Ernst, Frigyes.* **1932:** 1. *Alpár, László.* **1933:** 1. *Makai, Endre*; 2. *Felter, Károly.* **1934:** 1. *Vázsonyi (Weiszfeld), Endre.* **1935:** 1. *Bella, Andor*; 2. *Csanádi, György.* **1936:** 1. *Szele, Tibor*; 2. *Kepes, Ádám.* **1937:** 1. —; 2. *Frankl, Ottó.* **1938:** *Bodó, Zalán*; *Weisz, Alfréd.* **1939:** 1. *Sándor, Gyula*; 2. *Csáki, Frigyes.* **1940:** 1. *Hoffmann, Tibor*; 2. *Bizám, György.* **1941:** 1. *Ivancso, Imre*; 2. *Schweitzer, Miklós.* **1942:** 1. *Császár, Ákos*; 2. *Pál, Sándor.* **1943:** 1. *Ádám, István*; 2. *Takács, Lajos.*

First and Second Prize Winners of the Kürschák Mathematics Competitions (1947–1993)

1947: 1. *Pollák, György*; 2. *Ungár, Péter.* **1948:** *Czipszer, János*; *Neuwirth, Sándor.* **1949:** 1. —; 2. *Korány, Ádám*; *Róna, Péter.* **1950:** 1. —; 2. *Bognár, János*; 2. *Szekerka, Pál.* **1951:** 1. *Szekerka, Pál*; 2. *Kálmán, Lajos.* **1952:** 1. *Kálmán, Lajos*; *Kántor, Sándor*; 2. *Horváth, Ákos*; 2. *Nagy, Tibor.* **1953:** 1. *Surányi, Péter*; 2. *Vigassy, József.* **1954:** 1. *Vigassy, József*; 2. *Csiszár, Imre*; *Kovács, László.* **1955:** 1. —; 2. *Krammer, Gergely.* **1957:** 1. —; 2. *Makkai, Mihály.* **1958:** 1. *Kisvölcsey, Jenő*; 2. *Bollobás, Béla.* **1959:** 1. *Jelitai, Árpád*; *Halász, Gábor*; 2. *Muszélyi, György*; *Arató, Péter*; *Csanak, György.* **1960:** 1. *Mezei, Ferenc*; 2. *Bollobás, Béla.* **1961:** 1. *Kóta, József*; 2. *Bollobás, Béla*; *Molnár, Emil.* **1962:** 1. *Máté, Attila*; 2. *Kóta, József.* **1963:** 1. *Máté, Attila*; 2. *Fazekas Patrik.* **1964:** 1. *Gelencsér, László*; 2. *Corrádi, Gábor*; *Lovász, László.* **1965:** 1. *Makai, Endre*; 2. *Lovász, László*; *Pelikán, József.* **1966:** 1. *Hoffmann, György*; 2. *Elekes, György.* **1967:** 1. —; 2. *Babai, László*; *Mérő, László*; *Szeredi, Péter.* **1968:** 1. *Lemperth, László*; *Bajmóczy, Ervin*; *Mihaleczky, György.* **1969:** *Fiala, Tibor*; *Gönczi, István*; *Bajmóczy, Ervin*; *Csirmaz, László*; *Lukács, Péter*; *Nagy, András*; *Pintz, János.* **1970:** 1. *Bajmóczy, Ervin*; 2. *Ruzsa, Imre.* **1971:** 1. *Komjáth, Péter*; *Ruzsa, Imre*; 2. *Füredi, Zoltán.* **1972:** 1. *Tuza, Zsolt*; 2. *Győri, Ervin*; *Prőhle, Péter.* **1973:** 1. *Kertész, Gábor*; *Kollár, János*; 2. *Bacsó, Gábor.* **1974:** 1. —; 2. *Kollár, János*; *Neumann, Attila*; *Prőhle, Péter*; *Sparing, László.* **1975:** 1. *Miklós, Dezső*; 2. *Jakab, Tibor.* **1976:** 1. *Magyar, Zoltán.* **1977:** 1. —; 2. *Knébel, István.* **1978:** 1. —; 2. *Cseri, István*; *Csók, Tibor*; *Sali, Attila.* **1979:** 1. *Szegedy, Márió*; *Hajnal, Péter*; *Tardos, Gábor*; 2. *Bohus, Géza*; *Szegedy, Patrik*; *Varga, Tamás*; *Beleznay, Ferenc.* **1980:** 1. *Tardos, Gábor*; 2. *Bohus, Géza*; *Szabó, László.* **1981:** 1. *Tardos, Gábor*; 2. *Király, Zoltán.* **1982:** 1. —; 2. *Magyar, Ákos*; *Hetyei, Gábor.* **1983:** *Birkás, György*; *Erdős, László*; *Hetyei, Gábor*; *Kós, Géza*; *Magyar, Ákos*; *Pór, István*; *Szabó, Csaba*; *Törőcsik, Jenő.* **1984:** 1. *Erdős, László*; 2. *Szabó, Zoltán.* **1985:** 1. *Kós, Géza*; 2. *Csizmadia, György*; *Montágh, Balázs.* **1986:** 1. *Kós, Géza*; *Benczúr, András*; *Csom, Gyula*; *Montágh, Balázs.* **1987:** 1. *Drasny, Gábor*; 2. *Fleiner, Tamás*; *Keleti, Tamás*; *Tasnádi, Tamás.* **1988:** 1. *Fleiner, Tamás*; *Mándy, Attila*; *Sustik, Mátyás*; 2. *Bíró, András.* **1989:** 1. *Fleiner, Tamás*; 2. *Sustik, Mátyás.* **1990:** 1. *Pór, Attila*; 2. *Lakos, Gyula.* **1991:** 1. *Fleiner, Balázs*; 2. *Kálmán, Tamás*; *Katz, Sándor.* **1992:** 1. *Ujváry-Menyhárt, Zoltán*; 2. *Németh, Ákos*; *Tichler, Krisztián.* **1993:** 1. *Kálmán, Tamás*; 2. *Veres, Gábor.*

First and Second Prize Winners of the Károly Physics Competitions (1916–1943)

1916: 1. *Jendrassik, György*; 2. *Szilárd, Leó.* **1917:** 1. *Sztrókay, Pál*; 2. *Náray-Szabó, István.* **1918:** —. **1922:** 1. *Reguly, Zoltán*; 2. *Fábry, József.* **1923:** —. **1924:** 1. —; 2. *Körner, Endre.* **1925:** 1. *Teller, Ede*; 2. *Fuchs, Rudolf.* **1926:** 1. *Bakos, Tibor*; 2. *Hirka, László, Kurtz, Géza.* **1927:** 1. *Beke, Gyula*; 2. *Elek, Tibor.* **1928:** 1. *Gerő, Loránd*; 2. *Svéd (Schossberger), György.* **1929:** 1. *Párducz, Nándor, Székely, Lídia.* **1930:** 1. *Papp, György*; 2. *Gillemot, László.* **1931:** 1. —; 2. *Budó, Ágoston, Strausz, Antal.* **1932:** 1. —; 2. *Frőhlich, Károly, Lengyel, Sándor.* **1933:** 1. —; 2. *Makai, Endre*; *Kepes, Jenő, Tóth, László.* **1934:** 1. *Kisfaludy, János*; 2. *Erőd, János.* **1935:** —. **1936:** 1. *Zoltán, István*; 2. *Szélpál, István.* **1937:** 1. *Blahó, Miklós*; 2. *Gárdos, Pál.* **1938:** 1. *Gállik, István*; *2, Csiki, Jenő, Weisz, Alfréd.* **1939:** 1. —; 2. *Dolinszky, Tamás, Sándor, Gyula.* **1940:** 1. *Bizám, György*; *Freud, Géza, Hoffmann, Tibor*; 2. —. **1941:** —. **1942:** 1. —; 2. *Huszár, István*; *Kósa, Péter, Szablya, János.* **1943:** 1. *Vései, Miklós*; 2. *Kollányi, Miklós.*

First and Second Prize Winners of the Eötvös Physics Competitions (1949–1993)

1949: 1. —; 2. *Barabás, Pál*; *Holics, László*; *Koch, József*; *Könyves-Tóth, Gábor, Pócs, Lajos.* **1950:** 1.–2. *Mráz, József*; *Rozványi, Iván.* **1951:** 1. *Gulyás, Gábor*; 2. *Gyarmati, György.* **1952:** 1. — 2. *Szondy, Tamás.* **1953:** 1. *Zawadowski, Alfréd*; 2. *Kovács, László.* **1954:** 1. *Vigassy, József*; 2. *Siklósi, Péter.* **1955:** 1. *Bártfai, Pál*; 2. *Gutai, László*; *Klein, Tibor.* **1956:** 1. *Csiszár, Imre*; 2. *Rázga, Tamás*; *Zsombok, Zoltán.* **1957:** 1. *Papp, Kálmán.* **1958:** 1. *Széphalmi, Géza.* **1959:** 1. —; *2, Tusnády, Gábor*; *Magos, András.* **1960:** 1. —; 2. *Eisner, Gábor.* **1961:** 1. *Zakariás, László*; 2. *Fritz, József.* **1962:** 1. *Nagy, Dénes, Lajos*; *Szegi, András.* **1963:** 1. *Tichy, Géza*; 2. *Abos, Imre.* **1964:** 1. *Corradi, Gábor*; 2. *Doskar, Balázs.* **1965:** 1. *Gnädig, Péter*; *Juvancz, Gábor.* **1966:** 1. *Rácz, Miklós.* **1967:** 1. —; *2–3. Kádas, Sándor*; *Szalay, Sándor*; *Szűcs, Pál.* **1968:** 1. *Babai, László*; 2. *Marossy, Ferenc*; *Nagy, Zsigmond.* **1969:** 1. *Horváthy, Péter*; 2. *Lemperth, László.* **1970:** 1. *Blahó, Gábor*; *Harmat, Péter*; 2. *Bajmóczy, Ervin*; *Tichy-Rácz, Ádám.* **1971:** 1. *Nagy, András.* **1972:** 1. *Imhof, Gyula*; 2. *Gál, Péter.* **1973:** *Kertész, Gábor*; *Koltai, Ferenc*; *Meszéna, Géza*; *Simányi, Nándor.* **1974:** 1. *Ábrahám, Tibor*; *Vladár, Károly*; 2. *Szép, Jenő.* **1975:** 1. —; 2. *Szép, Jenő*; *Zimányi, Gergely.* **1976:** 1. *Vankó, Péter*; 2. *Kárpáti, Tibor*; *Surányi, Gábor.* **1977:** 1. *Neumer, Attila*; 2. *Kriza, György*; *Vankó, Péter.* **1978:** 1. —; 2. *Balogh, Elek*; *Lukács, József*; *Újházy, András.* **1979:** 1. *Kaufmann, Zoltán*; 2. *Bene, Gyula*; *Csordás, András*; *Frankhauser, József*; *Körmendy, Péter.* **1980:** 1. *Szalontai, Zoltán*; 2. *Umann, Gábor.* **1981:** 1. *Tokaji, Zsolt*; 2. *Mogyorósi, András*; *Petrovay, Kristóf*; 2. *Glück, Ferenc*; *Szállási, Zoltán*; *Tóth, Gábor.* **1982:** 1. *Csörgő, Tamás*; *Erdős, László*; *Tóth, Gábor*; 2. *Jeney, Tamás.* **1983:** 1. *Árkossy, Ottó*; *Erdős, László*; 2. *Fodor, Gyula.* **1984:** 1. *Kós, Géza*; 2. *Csillag, Péter*; *Czigler, Zoltán*; *Fáth, Gábor.* **1985:** 1. *Kaiser, András*; *Kós, Géza*; *Pfeil, Tamás*; 2. *Tasnádi, Tamás.* **1986:** 1. *Kaiser, András*; *Kohári, Zsolt*; 2. *Drasny, Gábor.* **1987:** 1. —; 2. *Gyuris, Viktor*; *Páczelt, Ferenc.* **1988:** 1. *Fucskár, Attila*; 2. *Csahók, Zoltán.* **1989:** 1. *Antal, Csaba*; *Rékasi, János*; 2. *Laux, Ádám*; *Somfai, Ellák*; *Szabó, Szilárd.* **1990:** 1. *Bodor, András*; 2. *Horváth, Tibor*; *Zóka, Gábor.* **1991:** 1. *Bodor, András.* **1992:** 1. *Katz, Sándor.* **1993:** 1. *Katz, Sándor*; 2. *Veres, Gábor*; *Prohászka, Zoltán*; *Gefferth, András*; *Futó, Gábor*; *Burcsi, Péter.*

Mathematical and Physical Journal for Secondary Schools (KöMaL) after the First World War

The journal did not appear during the first few years following the First World War. Fortunately, previous supporters of the journal worked enthusiastically on its rebirth. One of these supporters was *Andor Faragó* (1877–1944), a high school teacher, who in 1925 took over the editorship and publication of the journal until 1939.

The journal's title was *Középiskolai Matematikai és Fizikai Lapok* (Mathematical and Physical Journal for Secondary Schools). László Rátz lived to see the rebirth of the journal and greeted it with great joy. The problems and solutions section, which embodied the essence of the journal from its start, remained unchanged.

Since 1926 the journal has published the photos of its annual winners. Most of these young students became famous both nationally and internationally, among them Tibor Bakos, Zalán Bodó, Ágoston Budó, Pál Erdős, István Fenyő, Géza Freud, László Fuchs, Tibor Gallai, György Hajós, Endre Hódi, Ferenc Kárteszi, Elemér Nagy, János Surányi, György Szekeres, Tibor Szele, Pál Turán, Tamás Varga.

There was a brief intermission at the beginning of the Second World War due to finincial and political reasons, and the last edition appeared on May 15, 1939. The war had hardly finished when *Paula Soós* and *János Surányi*, two teachers from Szeged, distributed their makeshift journal, *Szegedi Ívek* (Szeged Pages), to the local schools. It gained recognition and demonstrated the need for a journal. What followed was the first edition of the third series of KöMaL at the beginning of November, 1947. The editors of the new series of the Középiskolai Matematikai Lapok were Paula Soós and János Surányi, and it was published by the János Bolyai Mathematical Society. Ever since then, the journal has appeared in ten issues per year.

In 1959, the Physics Column was started with the support of the Roland Eötvös Physical Society. In 1992, the journal again took on the name of the Középiskolai Matematikai és Fizikai Lapok (Mathematical and Physical Journal for Secondary Schools). The title of the journal changed often, but its goals always remained constant.

Every school year, the journal runs a points competition in the form of problems and exercises (6 + 8 in mathematics and 4 + 4 in physics in each edition). In addition to these there are easier mathematics problems, experimental physics problems, and difficult problems (4 in mathematics and 1 in physics). The best contestants in the points competition receive prizes and awards. Many of today's prominent mathematicians and physicists were at one time recognized as top contestants in the competitions; in fact, photos from their youth grace back issues of the journal.

The journal regularly reports on national and international competitions, prints articles on interesting results as well as original results in mathematics and physics, has sample problems for the college entrance exam, and book reviews. The journal is published today in 7000 copies. About 2500 students take part in the mathematics points competition and about 1500 for the physics points competition. More than 50,000 solutions are received yearly for more than 200 mathematics problems and 100 physics problems.

Viktor Scharnitzky

International Mathematics and Physics Olympiads

Competitions is a useful mean of searching for and discovering talent as well as encouraging and stimulating young people. Competitions in mathematics have a longer tradition than those in other subjects. In Hungary, the Mathematical and Physical Society started organizing mathematical competitions in 1894. Similar competitions in physics followed in 1916. The Ministry of Education has organized yearly competitions since 1923 in different subjects in the sciences, humanities, economics and many practical subjects. These competitions are very popular, as prizewinners are given preference when applying to the university. In the USSR the first competition in physics was held in 1939, in Poland in 1951 and in Czechoslovakia in 1959. Since then many other countries started national competitions, founding a mathematics competition first, and then a physics competition later.

The first international mathematical competition was called the 'International Mathematical Olympiad' (IMO) and was held in 1959 in Rumania. Since then it has been held every year. The Rumanian Mathematical Society invited seven countries to participate in the first contest. In the years following, more and more teams took place in the competitions. Last year's IMO was held in Istanbul, Turkey, and attracted 73 teams with 413 competitors from five continents.

International Physics Olympiads (IPhO) began later and developed much more slowly. Physics seems to be somewhat less popular with the various authorities. The first IPhO was in Warsaw in 1967 and came out of talks between Czechoslovakian, Hungarian, and Polish physicists (the three men principally responsible for the genesis of the competitions were, the Czech Rostislav Kostial the Hungarian Rudolf Kunfalvi, and the Pole Czeslaw Scislowski). The first competition included five countries, the three founding countries plus Bulgaria and Rumania, and consisted of teams of three students each. It was a happy little family event. The problems were not too difficult and there were excursions to Krakow, Gdansk and to the sea.

The second IPhO was held in Budapest in 1968 with eight participating countries: East Germany, the Soviet Union and Yugoslavia joined the existing five. In February 1969 an international committee drew up a plan for a statute for the competitions. It was accepted and is still in effect with only minor changes. The 24th IPhO was held in Williamsburg, Virginia, and included 196 competitors from 41 countries.

With few exceptions, the IMO and IPhO have occurred every year since they started, and the small group of countries responsible for their inceptions continues to shine at these competitions. The international status of the Physics Olympiad was recognized in 1990, when it received the "ICPE Medal for Physics Teaching" from the International Commission on Physics Education of IUPAP. The medal is transferred each year to a representative of the host country for the following year. (Similar medals were prepared for Prof. Rudolf Kunfalvi and the heirs of the other two principle founders.)

The teams usually consist of six students in mathematics and five students in physics. The majority are boys but occasionally one or two girls make their way onto the team. Students compete individually over the course of two days. In mathematics, the competition is purely theoretical, while in physics, there is also an experimental section. Hungarian students have participated in many IPhO's and, with the exception of 1978, all IMO's, and have consistently placed among the top. Last years IPhO was the most successful for our students: we received 3 gold medals, and placed second to Russia in the (unofficial) medal count. (These

Distribution of Prizes in 32 IMO's

Country	PV	I	II	III+HM	Σ	Σ/PV
Australia	78	8	15	28	51	0.65
Austria	158	6	20	63	89	0.56
Belgium	87	1	4	19	24	0.28
Brasil	83	4	–	19	23	0.27
Bulgaria	246	14	48	79	141	0.57
Canada	78	7	14	28	49	0.63
China	44	26	12	4	42	0.99
CIS	6	2	3	1	6	1.00
Cuba	109	–	2	14	16	0.15
Czech-Slovakia	229	10	50	80	140	0.61
Czech R.	6	1	2	3	6	1.00
Denmark	16	–	1	3	4	0.25
France	163	17	30	44	91	0.58
German D.R.	211	26	62	68	156	0.74
F.R.Germany	90	24	35	25	84	0.93
Germany	18	5	11	2	18	1.00
Great Britain	182	23	46	69	138	0.76
Hungary	234	49	92	65	206	0.88
Hong Kong	30	–	3	11	14	0.47
India	24	1	5	13	19	0.79
Iran	43	4	15	12	31	0.72
Israel	72	3	11	26	40	0.56
Japan	24	1	10	6	17	0.71
North Korea	12	–	4	5	9	0.75
South Korea	42	1	6	16	23	0.55
Luxemburg	19	2	3	7	12	0.63
The Netherlands	160	2	16	30	48	0.30
New Zealand	36	–	1	9	10	0.28
Norway	52	–	5	10	15	0.29
Poland	227	10	34	77	121	0.53
Romania	242	36	69	76	181	0.75
Russia	12	6	3	3	6	1.00
Singapore	36	–	5	15	20	0.56
Slovakia	6	1	3	1	5	0.83
Soviet Union	204	77	67	47	191	0.94
Sweden	181	4	19	40	63	0.35
Switzerland	8	–	1	2	3	0.37
Taiwan	12	1	7	3	11	0.92
Thailand	30	–	2	6	8	0.27
Ukrain	11	–	2	8	10	0.91
USA	126	31	61	24	116	0.92
Viet-Nam	102	11	37	41	89	0.87

Distribution of Prizes in 24 IPhO's

Country	PV	I	II	III	HM	Σ	Σ/PV
Australia	35	–	1	10	15	26	0.74
Austria	35	–	1	10	15	26	0.74
Bulgaria	117	7	11	24	36	78	0.67
Canada	45	1	–	9	9	19	0.42
China	40	15	12	9	2	38	0.95
Croatia	10	–	–	–	4	4	0.40
Czecho–Slovakia	117	14	24	29	27	94	0.80
Czech R.	5	2	2	–	1	5	1.00
Finland	75	1	1	11	25	38	0.51
France	35	2	5	9	14	30	0.86
German D.R.	99	8	14	35	27	84	0.85
F.R.Germany	75	8	20	25	11	64	0.85
Germany	15	2	2	7	3	14	0.93
Great Britain	50	5	14	12	11	42	0.84
Hungary	117	21	19	43	24	107	0.91
Italy	45	–	–	6	7	13	0.29
Iran	19	–	–	6	4	10	0.52
Korean R.	15	–	–	4	2	6	0.40
Lithuania	15	–	–	4	2	6	0.40
The Netherlands	60	3	8	15	20	46	0.77
Norway	50	1	–	3	7	11	0.22
Poland	117	14	21	31	33	99	0.85
Romania	117	18	29	33	19	99	0.85
Russia	10	6	3	1	–	10	1.00
Singapore	25	1	–	4	7	12	0.48
Slovakia	5	–	1	1	1	3	0.60
Slovenia	7	–	–	2	1	3	0.43
Soviet Union	104	41	26	23	12	102	0.98
Sweden	80	1	8	12	28	49	0.61
Turkey	40	1	–	5	6	12	0.30
Ukraine	10	2	5	2	–	9	0.90
USA	40	5	6	12	8	31	0.78
Viet-Nam	50	–	–	9	11	20	0.40
Yugoslavia	79	1	7	14	10	32	0.41

Explanations:

PV — „Participation Volume", i.e. the total possible number of participants (from a given country) in the Olympiads which the country took part

I, II, III — number of prize winners (In the 1st IPhO there were 7 prize winners classified linearly. As their scores were very high and very close to each other, in the above Table all the winners of the 1st IPhO are treated as first prize winners.)

HM — number of the participants who obtained the Honourable Mentions

Σ — number of the participants who passed the competition successfully (I+II+III+HM) results are of particular interest if one considers that our educational systems are struggling with financial problems.) We can judge the performance of our students by the table of medals. The adjoining two tables also show normalized results (based on how many events a competitor took place in), by which we can better examine the results.

From the instructional point of view, the Hungarian students may have the following to thank their strong performance: an elevated level of instruction, many competitions in both math and physics, preparatory classes for the contests, and not last, KöMaL's points contest. The journal plays two main roles: it helps close the gap between mathematics and physics in the real world and that taught in school and it also serves to awaken the students' interest. From the point of view of the olympiads, the journal is most helpful in teaching the students how to write mathematics (and physics) correctly. (This is a skill that is completely lacking from the schools, but is a determining factor in the olympiads.) The KöMaL points contest and the competitions which the students take part in help us to prepare for the olympiads as well as to choose the best possible team.

István Reiman **Péter Gnädig**

Remembering the student years

Many prominent Hungarian physicists and mathematicians achieved their famous results abroad, but they remembered their school years, the student journal and student competitions which contributed to their careers.

János (John von) Neumann was the mathematician of the 20th century with perhaps the greatest impact beyond mathematics. Born in Budapest in 1903, he laid the mathematical foundation of quantum mechanics in his twenties, in Göttingen. He was an honorary member of the Eötvös Society. He became professor at the Institute for Advanced Studies in Princeton, and went on to design the electronically programmable computer. He developed the theory of games. He received the first Fermi Medal, and later the Freedom Medal.

"Johnny entered the Lutheran Gymnasium in 1914, when the scholastic achievements of Budapest gymnasia were at their peak. László Rátz was his math teacher, and Wigner's as well. Jenő (Eugene) Wigner was asked in the late 1970's (60 years after leaving the Lutheran Gymnasium): 'Do you remember Rátz?' Wigner answered: 'There he is!' and pointed to a picture of Rátz on his office wall. Rátz's recognition of Johnny's mathematical talents was instant. He paid a visit to Max von Neumann, Johnny's father, and told him that it would be nonsense, and perhaps sinful, to provide the boy with only the conventional education in mathematics that the Lutheran Gymnasium, and Rátz himself, could offer, as excellent as it might have been. Rátz proposed instead to take responsibility for seeing that a great deal more was provided, if there were no parental objection. There was no educational price to be paid. Max agreed at once. Rátz turned his student over to the mathematicians at Budapest University, themselves men of no small renown. Prof. Kürschák asked *Gábor (Gabriel) Szegő*, to give some university teaching to the lad. Before he finished high school, Johnny had been accepted by most of the university mathematicians as a colleague. Johnny's first published paper was sent for publication when he was seventeen."
(*John von Neumann; by Norman Macrea, Pantheon Books, NY1992.*)

Gábor Szegő

Gábor (Gabriel) Szegő (1895–1985) was a student of Lipót Fejér, later a special tutor of the high-school student János Neumann, and eventually became a professor at Stanford University. At Stanford, he was joined by the Hungarian *György (George) Pólya*, and together they had a large impact on Silicon Valley. In the 1950's he initiated mathematical competitions in California following the pattern of the Eötvös competitions.

"[For a successful mathematics competition] some sort of preparation is essential to arouse public interest. In Hungary, this was achieved by a (high-school mathematics) journal... I remember vividly the time when I participated in this phase of the Journal (in the years between 1908 and 1912). I would eagerly await the arrival of the issue each month and my first priority was to look at the problem section, almost breathlessly, and to start grappling with the problems without delay. The names of others who were in the same business were quickly known to me, and frequently I read with considerable envy how they had succeeded to solve some problems which I could not handle with complete success, or how they had found a better solution (simpler, more elegant, or wittier) than the one I had sent in."
(*Hungarian Problem Book, by E. Rapaport, Random House, New York, 1963.*)

Jenő (Eugene) P. Wigner was born in Budapest in 1902, and received the Nobel Prize in Physics as a professor at Princeton University in 1963 for understanding the role of symmetries in quantum mechanics, for the discovery of parity, and for applying quantum mechanics to atomic nuclei. He is an honorary member of the Eötvös Society and the Hungarian Academy. He received the Fermi Award in 1958, and the Atoms for Peace Award in 1960 from President Eisenhower. In his acceptance speech for his Nobel Prize, he said:

"There were many superb teachers at the Lutheran gymnasium. But the greatest was my mathematics teacher László Rátz. Rátz was known not only throughout our gymnasium but also by the church and government hierarchy and among many of the teachers in the country schools. I still keep a photograph of Rátz in my workroom because he had every quality of a miraculous teacher: He loved teaching. He knew the subject and how to kindle interest in it. He imparted the very deepest understanding. Many gymnasium teachers had great skill, but no one could evoke the beauty of the subject like Rátz. Rátz cared deeply about mathematics as a discipline. He edited a mathematical journal for secondary schools which came to be read

all over the nation. He ran the journal for 20 years, often distributing it with his own money. — At the retirement of Imre Gobi, the gymnasium director, the staff named Rátz as his successor. That gave Rátz a formal title and likely a higher salary. Most men would have said, 'Thank you kindly for the promotion. This is very fine.' But Rátz worried that his new duties would hurt his teaching. He knew how much energy is needed to evoke the deepest beauties of mathematics. And after five years of distinguished service as director, Rátz quietly resigned as director and became just a teacher again. — He took special care to find his better students and to inspire them. Rátz felt so privileged to tutor a phenomenon like Neumann Jancsi that he refused any money for it. You might say, 'Well, von Neumann was one of the great mathematicians of our century. Of course he deserved private classes as a boy.' But look at this from the teacher's point of view. He appeared to be a genius. But, of course, not yet famous at all. His brain wasn't that of an adult. He had never published. He was just a startling 10-year-old boy, working next to 20 other bright 10-year-olds. Who could know that this precocious 10-year-old would someday become a great mathematician? Somehow Rátz knew. And he discovered it very quickly. — Rátz was just as nice to me and nearly as devoted as he was to Neumann. Rátz was the only gymnasium teacher to invite me into his home. There were no private lessons. But Rátz lent me many well-chosen books, which I read thoroughly and made sure to return in good condition. — Rátz also compiled for his students a book of *common sense* mathematical problems. I solved a few of them, but most I found fantastically hard. Often in the years since, when I have been in no mood for work, I have taken Rátz's little book from the shelf and studied those common sense problems."
(*The Recollections of Jenő (Eugene) P. Wigner, as told to Andrew Szanton. Plenum, New York, 1992.*)

Ede (Edward) Teller was born in Budapest in 1908. He is famous for the understanding and realization of thermonuclear reactions. He received the Fermi Award in 1962. He is an honorary member of both the Hungarian Academy and the Eötvös Society.

"Knowledge was pursued vigorously in the gymnasia, including the one founded by Móric (Moritz von) Kármán, father of Tódor (Theodor von) Kármán. This gymnasium was called Minta, roughly translated as 'model' which is to say it was a demonstration school for the teachers. — In the spring of 1918 Edward Teller completed his four years in the Mellinger Primary School. His parents had to decide where he would go for the rest of his education. The decision of Edward's parents was made after rejecting two other possibilities. The first was the school operated by the Catholic Piarist Order, but those who attended were expected to become Christians. The second possibility was the Lutheran school, which included Jenő (Eugene) Wigner and John von Neumann among its students. Finally his parents agreed that the Minta was a better choice for Edward. He went off to the gymnasium a trifle timid. He became an enthusiastic reader, with a particular passion for the vintage science fiction of Jules Verne. In the school he enjoyed everything except Latin. His particular problem, however, was mathematics, toward which, from the start, he was apathetic. Predictably, the cause of the apathy was not ignorance but boredom. He was far ahead of his classmates. Eventually, the school exempted him from the study of mathematics during his first year, but, before this wise decision was rendered, there was considerable hardship for Edward. It is understandable that the ultimate humiliation for a teacher is to be outshone by one of his pupils. That is precisely what happened to Dr. Oberle, who was not only Edward's mathematics teacher but principal of the Minta. Oberle had worked out an algebraic equation on the blackboard to his own satisfaction and presumably to the satisfaction of the forty-odd boys in the class. When he turned around, the ten-year-old's hand was raised: 'Is there something wrong, Teller?' he asked sarcastically. Edward suggested there was a better way of doing it. 'Then come up here and do it,' said the irritated teacher. Edward did, and with more finesse than his teacher. Oberle's response was to the point: 'Are you a genius, Teller? Well, I don't like geniuses.' — Edward was about fourteen when he acquired a book on Einstein's special and general theory of relativity. His physics teacher, however, took it away from him and said he wasn't ready for it yet. Edward, after plowing through these theories that many physicists could not understand, conceded that his teacher was right. Later, after he had completed his course and passed the final exam, the teacher returned the book. 'Now you are ready', said the teacher, and once again he was right."
(*Energy and Conflict [the life and times of Edward Teller], by Stanley A. Blumberg — Gwinn Owens G. P. Putmann, New York, 1976, page 15.*)

Pál (Paul) Erdős was born in Budapest in 1913 in Budapest. He was the son of two high school mathematics teachers who were good friends of Andor Faragó. Paul Erdős won the Wolf Prize in 1984, a prize in mathematics which has the same prestige as the Nobel Prize in other sciences. He is an honorary president of the János Bolyai Society, member of The National Academy of the U.S.A., The Royal Society of London, American Academy of Arts and Sciences, Australian Academy, Dutch Royal Academy, Hungarian Academy of Sciences, and the Indian Academy of Sciences.

the young Pál Erdős *Pál Erdős today*

"He is probably the most profilic creative mathematician alive today. He is an indefatigable traveller, who disseminates mathematics and mathematical problems all over the world. With over 1500 articles or books, he has collaborated with many more mathematicians than anyone else, past or present.

The influence of Paul Erdős on today's mathematics and today's mathematicians stems not only from the vast body of results we owe to him, but also, and most strongly, from the great number of remarkable mathematical problems he creates and disseminates. They come from number theory, graph theory, geometry, set theory, and they range in difficulty from ingenious high school competition problems to the most difficult research problems that defy, and will continue to defy for many years to come, all attempts at solution; their common feature is that they are all fascinatingly interesting.

The World Federation of National Mathematics Competitions in 1991 established the David Hilbert Intrenational Award and in 1992 the Paul Erdős Award. Both awards recognise contributions of mathematicians which have played a significant role in the development of mathematical challenges which have been a stimulus for the enrichment of mathematics learning; the former at the international level and the latter at the national level."
(*Paul Erdős, by Peter J. O'Halloran, Journal of the World Federation of National Competitions Vol. 5 No. 1, June 1992*)

"— Do you feel that your mathematical development was affected by the high school mathematics newspaper (Középiskolai Matematikai Lapok)?

— Yes, of course. You actually learn to solve problems there. And many of the good mathematicians realize very early that they have ability."
(*A visit to Hungarian Mathematics, Reuben Hersch, Vera John-Steiner, The Mathematical Intelligencer Vol 15. No. Springer-Verlag, New York, 1993.*)

"— The great flowering of Hungarian mathematics — to what do you attribute this?

— There must be many factors. There was a mathematical journal for high schools, and the contests, wich started before Fejér. And once they started they were self-perpetuating to some extent. Hungary was a poor country — the natural sciences were harder to pursue because of cost, so clever people went into mathematics. But probably such things have more than one reason. It would be very hard to pin it down."
(*Mathematical People, by D.J. Albers and G.J. Alexanderson, Birkhauser, Boston (1985) Interwiew with P. Erdős*)

Tibor Gallai

Tibor Gallai (Grünwald), was born in 1912 in Budapest. As a winner of the prestigeous Eötvös Mathematical Competition he was admitted to the university. Gallai received his diploma as a high school

teacher in mathematics and physics in 1936, but only after the Second World War did he have a chance to fully practice his beloved profession. He became a teacher in a Jewish girls' high school in Budapest. Out of the 26 students in his first class, three subsequently became research mathematicians. In 1949, Gallai became professor of mathematics at the Technical University of Budapest. In 1958 he joined the mathematical Institute of the Hungarian Academy of Sciences where he worked until 1968. He had a major influence on graph theory and combinatorial optimization.
(*Tibor Gallai, 1912–1992., by László Babai and Vera T. Sós, Combinatorica 14 (1994) 371–372.*)

"Nowhere else in the world is there this kind of high school paper, and this more than anything else is responsible for the excellence of Hungarian mathematics."

(*Visit to Hungarian Mathematics, by Reuben Hersh, Vera John-Steiner, The Mathematical Intelligencer Vol 15. No 2. Springer-Verlag New York 1993.*)

György (George) Szekeres is a chemist–mathematician now living in Australia. He is a member of both the Hungarian and Australian Academies of Sciences.

György Szekeres   Eszter Klein

"I didn't know about KöMaL until the beginning of my last year of high school when my physics teacher Károly Novobátzky (later professor of theoretical physics at Budapest University) called my attention to the journal. I immediately became and avid problem solver. It was only then that I realized that I have a talent for mathematics. By that time I had already decided to take up chemical engineering, so I enrolled at the Technical University. However, soon after finishing high school I got acquainted with many of my fellow KöMaL problem solvers whom until then I had only known from the pages of the journal: Pál Turán, Pál Erdős, and Tibor Grünwald-Gallai, among others, who later made such a deep mark on Hungarian and world mathematics. As a result, even during my chemistry studies I never lost contact with mathematics and read avidly any mathematical book that came my way (for instance I read through Landau's three-volume book *Number Theory*). By the time I got my chemical engineering degree I already wrote a mathematical paper with Turán. Soon afterwards my second paper was born, this time with Erdős, in which we solved a combinatorial-geometrical problem proposed by another former KöMaL problem solver, Eszter Klein. This paper eventually became rather well known as one of the pioneering works on what is now called Ramsey theory. For me it had another significance: Eszter became my wife in 1937. Erdős called it afterwards the 'Happy End Problem'."
(*KöMaL centennial issue, December 1993.*)

János (John G.) Kemény was born in Budapest in 1926 and emigrated to the USA with his family in 1940. There he was a student of John von Neumann, and later an assistant of Albert Einstein's in Princeton. In Los Alamos he worked with computers under Richard Feynman. Convinced that computers are not only for specialists but for everyday people too, he created the BASIC language, now the most often used computer language in the world. For developing the time sharing system for networking computers, he was awarded the first Robinson Prize in 1991 by IBM. He became well-known for heading the presidental commission investigating the TMI nuclear power station accident in 1979. He was an honorary member of the Eötvös Society. In an interview he recalled:

"Why are so many great mathematicians in Hungary? There are a few fields in which there are an inordinate number of Hungarians: mathematics, theoretical physics and Hollywood. I forget who the producer in Hollywood was, with a big sign on his desk saying: 'Being a Hungarian is not enough; you must have talent.' I don't quite believe Gail Young's theory that the Hungarian language is so hard that only the brightest children manage to survive. Certainly, for mathematicians and theoretical physicists, the school system in Hungary was very good. — Let me give you an example. I went through 7 and a half grades of school in Hungary. I had a mathematics teacher who would have been well qualified to teach at a good college. He just did an enormous amount to strengthen my interest in mathematics. I liked mathematics before that. But being interested and knowing something is very different. This teacher was better than any teacher I had in high school in the United States, really significantly better. — There was a mathematical contest for high school seniors in Hungary that was a very big thing. If you were talented, I am told, all through your last three years you would drill on practice tests. It was a great honor, not just for yourself, but for the school. — There is another interesting story about that teacher. When we left for the United States [in 1940], my whole class came out to the train to see me off, and my math teacher did too. It was really nice. He said something that has stuck with me all the time. He said that he was terribly happy for me that I was leaving for the United States, because he was worried about the future of Hungary. On the other hand, he said, he had only one regret. He never had a winner in the math competition. Look, for God's sake, I was four and a half years away from the exam, and he was already thinking that maybe I could make it to the top in the competition..."

(*John G. Kemeny, interviewed by Lynn A. Steen in 1980. Mathematical People — Profiles and Interviews, edited by D.J. Albers and G.L. Alexanderson, page 153. — Fizikai Szemle 1993/5.*)

Péter Lax was born in 1926 in Budapest. He left for America at the age of 15 where he was lucky to fall into the hands of John von Neumann. He later became head of the Courant Institute and President of the American Mathematical Society. For his work, he received the Wolf Prize in 1987 and was elected an honorary member of the Hungarian Academy of Sciences in 1994. In an interview with the KöMaL, he said:

"When we arrived in America at the end of 1941, John von Neumann, who had been told by Dénes König that I was coming, came to our house to greet us and spent an entire afternoon with me. This was very kind of him, especially if you consider, and I just found this out later, that he was very busy with his work for the war effort. Fate later had it that we spent many summers together at Los Alamos. He had an enormous effect on my life. I believe that some people are born with a gift for mathematics, just as in music. It is very easy to recognize the difference between someone who is tone deaf, but interested in music, and someone like Mozart, Schubert, or Mendelsson, who already at an early age, demonstrates that he understands music in an entirely different way. Of course, intellectual development is also important for any talent. It was certainly very important for Mozart that his father traveled with him to many foreign places and exposed him to different types of traditional music. I was very fortunate in this respect as I started with the Hungarian tradition, having *Dénes König* and *Rózsa Péter* among my mentors, and followed this with the German tradition in America. In Los Alamos I was exposed to the physics tradition and was lucky to have the opportunity to work with computers from their infancy.

"Mathematics is a very rich field that requires a life of study, and the earlier we start, the better. Only reading or studying does not fit in well with most people's temperaments, and is not sufficient. As mathematics is not passive, but rather active, I have the most reverence for those problems that need certain theoretical concepts in order to solve them. The problems can be very difficult, they can look impossible to solve, but if a solution is found, it is unbelievably worthwhile. Of course these are not problems that are out of reach for high school students. I find that there is a tradition of problems that are appropriate for high school students and then there are those that are too difficult. We have to reformulate those that appeared too difficult yesterday, and we will see that we are able to explain them so that they are understood by high school students. A journal should have articles like these."

Ferenc Mezei was born in Budapest in 1942. He studied and worked in Budapest. Now he is professor in Berlin and head of the Scientific Council of the Hahn-Meitner Institute. In 1984 he received the Hewlett-Packard Prize of the European Physical Society for the invention of the neutron spin echo method for studying solids and macromolecules. He received the Eötvös Medal of the Eötvös Society. He recalled:

Ferenc Mezei

“When I was enrolled in the Rákóczi Gymnasium (grade 9), at the suggestion of my enthusiastic teachers I ordered the Középiskolai Matematikai Lapok, but I put them aside without reading them. During the summer holiday, on rainy days I began to solve the problems in the Journal. After one month I counted my score. It turned out that I would have won the school-year-long problem solving competition, had I mailed the solutions back to the Student Journal. The next school year (grade 10) I mailed the solutions regularly to the Journal. For this reason I missed one day from school each month. My summer exercise was certainly helpful: I won the problem-solving competition far ahead of other students! So I had been seduced. I only had to decide: should I become a mathematician or a physicist? In the end, it was the hard facts and unexpected traps of measurements that attracted me to experimental physics. Nature treats benevolently those who keep questioning her respectfully, and take the occasion to explore one or two of her secrets.”

(Ferenc Mezei won the National Mathematics Competition in 1959. He finished first in the Kürschák Mathematics Competition and second in the Eötvös Physics Competition in 1960. In the same year he received a first prize at the International Mathematics Olympiad.)

László Lovász

László Lovász became an internationally renowned mathematician at a very young age. Ha was elected to the Hungarian Academy of Sciences in 1985. Presently he is a Professor at Yale University.

“The Math-journal is of course more than just a points contest. The first issue I saw had an article by Paul Erdős about combinatorial geometry. I was pleasantly surprised that I could understand the ideas of the big mathematicians, at how many beautiful, difficult, unsolvable questions there are if one just looks around, and all in such a classical setting as geometry. I read the article at least 20 times. Paul Erdős's articles and lectures today still contain the same magical power. I am very happy for the KöMaL that it reaches so many students.”

(*KöMaL centennial issue in Hungarian, December 1993.*)

Assembled by **George Marx**

Problems of Paul Erdős

1. From a point inside a triangle, we call the distances to the vertices a, b, and c and the distances to the edges x, y, and z. When does the quotient

$$\frac{a+b+c}{x+y+z}$$

take its minimum?

2. For a set of n points in the plane, determine the maximum number of times the unit distance can appear. Give upper and lower bounds for this quantity. ($cn\log n$ is a lower bound.)

Küldte: *From an article of Erdős and Purdy*

3. Prove that

$$\sum_{n=2}^{\infty} \frac{1}{n!-1}$$

is irrational.

4. Given a convex n-gon in the plane, let s_1, s_2, ... be the number of times each distance appears, so $\sum\limits_{n=1}^{\infty} s_i = \binom{n}{2}$. Prove that for a convex polygon,

$$\sum_{n=1}^{\infty} s_i^2 < cn^3,$$

for some constant c; and for $n > 8$ that the sum is maximum for the regular polygons.

Küldte: *A problem of Erdős and Fishburn*

5. Is it true that every convex polygon has a vertex v without 4 vertices equidistant from v? (For 3, there are counterexamples.)

6. Let $1 \leq a_1 < a_2 < \ldots < a_{n+2} \leq 3n$ be $n+2$ integers. Prove or disprove that there always are three of them $a_i < a_j < a_k$ for which $(a_j + a_k)/a_i$ is an integer (i.e. the sum of the two larger one is a multiply of the smaller one). If the statement is true, show that the $n+1$ numbers $2n$, $2n+1$, ..., $3n$ do not have this property, and, therefore, the statement cannot be improved.

Küldte: *A problem of Erdős and Sárközy*

7. Denote by $f(r)$ the maximum value for the sum of the sidelength of r squares, inscribed in a unit square so that their interiors do not intersect. It is easy to see that $f(k^2) = k$. Prove or disprove that $f(k^2+1) = k$ $\quad$ ($f(k^2+2) > k$ is easy).

Anyone who solves any of the above problems should send the solutions to: Paul Erdős, MTA Mathematical Institute, Budapest, P.O.B. 127, H–1364, Hungary

Problems of Frigyes Károlyházy

1. A fluff of negligible mass is tied to a thin thread at the end of a whip-handle. One end of the whip-handle is at rest, the other end moves around a circle. What is the path described by the fluff?

2. There is some liquid in equilibrium in a U-shaped tube. A ball of very large mass is placed under the left arm. How do the liquid levels change?

3. There is water in a dish placed in the middle of the disk of a turntable. A ping-pong ball is floating on the water. What happens to the ping-pong ball when the turntable is turned on?

4. A little something is standing still in point A at the edge of a level board. We push it in such a way that it reaches point B. It actually gets there in 2 seconds. The distance between A and B is 1 meter. Does this little something have wheels?

5. A chest with a mass of 1 kg, that does not slip by itself, is pulled up and down a slope. The total amount of work done is 10 J. What is the maximum height h of the slope?

6. A glass vessel contains mercury, and a glass capillary tube hangs into it. We connect the mercury to a high potential with reference to the ground. Does the mercury in the capillary tube move, and if it does, in what direction?

7. An electrically neutral, closed, hollow metal spherical shell, floats in outer space. A small, positively charged metal ball is at rest in its centre, free. A homogeneous electric field is created around the spherical "Faraday cage" by means of remote capacitor plates. Will the little metal ball or the spherical shell start moving, and if so, how?

8. There is a grounded metal pin in the vicinity of one of two neutral metal spheres. An electrically charged sphere is slowly lowered from above, in the middle between the two spheres. Where will a spark first strike?

9. Flat coil A is lying in a plane. In the first case, the magnet M perpendicular to the plane of the coil, according to *Fig. a)*. In the second case, according to *Fig. b)*, there is an identical coil B, situated symmetrically in the plane. The bar magnet is suddenly pulled up. In what case will the current induced in coil A be larger?

LENINGRAD MATHEMATICAL OLYMPIADS 1987–1991

by Dmitry Fomin and Alexey Kirichenko

The Leningrad Mathematical Olympiad is perhaps a unique phenomenon in the world of mathematical competitions. First, unlike the majority of other contests, almost all the problems proposed are new and original. The contest creators have been directing their best efforts towards this goal for the past 60 years. Second, this is the only official competition in Russia (and perhaps in the whole world) that is held in oral form.

"*The Leningrad Olympiads are among the most exciting and interesting mathematical events in the world on the pre-college level.*" — Mark Saul

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The Leningrad Mathematical Olympiad is the eldest (founded in 1934) mathematical competition in Russia. The authors of this book have been involved with this competition for many years. The book includes complete solutions to all the problems and competition statistics.

Dmitry V. Fomin is Assistant Professor at the Chair of Higher Geometry on the Faculty of Mathematics and Mechanics at the St. Petersburg State University. He has much experience in educating able high-school students. He is also an executive secretary of the Leningrad Mathematical Olympiad's jury and member of the problems committee of the International Tournament of the Towns.

Alexey L. Kirichenko is a Visiting Lecturer of Mathematics at the Leningrad Electro-Technical University, teaching courses for high-ability undergraduates. He also consults in the area of computer graphics. For several years, he was a member of the Leningrad Mathematical Olympiad's jury and ran the math courses for gifted pupils during the years 1985–1990.

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How to find the largest element by comparisons

Of n tennis players, how many games are needed to find the best? Given n bodies of different weight, how many comparisons are needed to find the second heaviest if one can only use a beam balance? These and similar questions are the basic problems of the theory of selections, an important chapter of computer science. For the answer we have to produce an algorithm, i.e. a plan to execute the comparisons, and, as we are interested in optimal algorithms, we must be able to show that no shorter algorithm may work.

The theory of computation uses the following model: We are given n data, $A_1, \ldots, A_n$, which are in a certain order. This order is not known to us, but we can find out the relative order of any two elements by comparing them. The task is to find the largest, the second largest, etc. by the least number of comparisons. We could assume that $A_1, \ldots, A_n$ are real, or even natural numbers, as is indeed the case in some sports application, but we should not do that, as it is impossible to learn the values of the data by comparing them. In an abstract setting, there is an unknown relation among the data, possessing the familiar properties of the "less than" relation, i.e. it is transitive and trichotomic. In the following proofs we will use some expressions naturally arising from sports.

Fig. 1

Fig. 2

Theorem 1. Of n data one can find the largest using $n-1$ comparisons.

Theorem 2. To find the largest among n data one always needs at least $n-1$ comparisons.

Proof of Theorem 1. We give two algorithms. In the first (see *Fig. 1*) we first compare A_1 and A_2 then the winner with A_3, etc. This is the method of challengers, in which the winner stays (at the table, on the tennis court, etc.)

The other method performs the comparisons in rounds (see *Fig. 2*) As before, the number of games is $n-1$, as all but one participant loses in some game, and then leaves. (To calculate the number of games directly is a little harder. Let $f(x)$ be the number of games for x participants. Then $f(1)=0$. If $x>1$ and $2y \le x \le 2y+1$, then there are y games in the first round. Then $x-y$ participants survive. Consequently $f(x)=y+f(x-y)$. Using induction, we get $f(x)=x-1$.)

Proof of Theorem 2. All but one participant (the winner) must lose a game and there is one loser per game.

To formulate the following result let r be the unique natural number satisfying $2^{r-1} < n \le 2^r$. Then there are r rounds in *Fig. 2*, and the winner plays exactly r games.

Theorem 3. One can find the second largest entry among n data using $n+r-2$ comparisons ($n>1$).

Proof. We first find the largest element by $n-1$ comparisons, using the method of *Fig. 2*. All the other elements have lost sometime. The second largest element could only have lost from the largest; consequently it is one of the r elements beaten by the winner. It can be found by $r-1$ further comparisons, using either method of Theorem 1. They sum up to $n-1+r-1=n+r-2$ comparisons.

Theorem 4. No algorithm can always find the second largest element using fewer than $n+r-2$ comparisons.

Proof. It is not true, as in Theorem 2, that we always need at least $n+r-2$ comparisons. With luck, the method of *Fig. 1* can work with $n-1$ comparisons if the challenger always wins.

Assume that some algorithm has determined S, the second largest element. We do know that S is not the largest one, so it has lost in a comparison (otherwise it could claim that it was the largest). But it could only be beaten by the largest element, L. Therefore we know that $L > S$, and then we know the identity of L. Theorem 2 requires $n-1$ comparisons.

Let p be the number of comparisons in which L has participated. There are, therefore, p elements, beaten by L, and one of them is S. As the algorithm decides which is S, we know for sure that $p-1$ of them are not S. This is only possible if they are beaten by someone else, too. In total, there are $(n-1)-(p-1) = n-p$ elements beaten once, and $p-1$ other elements beaten at least twice. The number of games, i.e., the number of losers is at least $(n-p)+2(p-1) = n+p-2$. We are done, if we can guarantee $p \geq r$.

To complete the proof, we are going to show that for every algorithm there is an order of the data which forces $p \geq r$. Every step in the algorithm is an $A_i - A_j$ comparison. As we try to make L participate in as many comparisons as possible, it is reasonable to demand that the participant who participated in the larger number of games should win.

The rule is as follows: When A_i and A_j meet, we may assume that $A_i > A_j$ and $A_i < A_j$ are both possible, otherwise the comparison would not give any extra information and we could leave out this step. If A_i and A_j are both previously unbeaten, we choose as winner the one which has played the larger number of games, (if they have played an equal number, then either of them). If A_i is unbeaten and A_j is beaten, let $A_i > A_j$. If both are beaten, either may win.

We must prove that these selections guarantee $p \geq r$. Assume that at the beginning of the tournament every player gets a card and when he is first beaten, he passes the collection of cards he has collected so far to the winner of that game. During the tournament the cards keep moving. They are always in the hands of the unbeaten players, and eventually L collects them all.

We show that the following statement is always true. If, at a certain stage, the unbeaten A_i has played x games, then he has at most 2^x cards.

We can prove this by mathematical induction on x. This is true for $x = 0$; if A_i has not played any games, he has only one card. If A_i meets A_j he can extend his card collection only if he beats the previously unbeaten A_j. If A_j has played y games, $x \geq y$ holds by our rule for $A_i > A_j$. By the inductive hypothesis, A_j passes at most 2^y cards (all he has) to A_i, who now has played $x+1$ games and has at most $2^x + 2^y \leq 2^x + 2^x = 2^{x+1}$ cards. Consequently, the above statement is proven by induction.

At the end of the tournament every card is in the hands of L, so we have $2^p \geq n > 2^{p-1}$, therefore we have $p \geq r$.

There is one more instance where it is possible to give a sharp bound on the number of comparisons. Let x be the natural number satisfying $2x+1 \leq n \leq 2x+2$.

Theorem 5. One can find among n data both the largest and the least elements by $n+x-1$ comparisons.

Proof. With the method of Fig. 2 we can find the largest element by $n-1$ comparisons.

The least element cannot be among the winners of the first round, so it is among the $x+1$ other elements. By Theorem 1, it can be found by x further comparisons.

Another possibility is to use induction from n to $n+2$. The theorem obviously holds for $n = 1$ or 2. Assume that with the help of $n+x-1$ comparisons we have found the largest and the least elements among $A_1, \ldots, A_n$, and two new elements, A_{n+1} and A_{n+2} are added. With one comparison we find P, the larger, and Q, the smaller of A_{n+1} and A_{n+2}. Then, by comparing P with L and Q with M, where L and M are currently the largest and least elements, we get the required elements. Adding these three comparisons, we proved the result for $n+2$.

Theorem 6. No algorithm can always find both the largest and the least elements with less than $n+x-1$ comparisons.

Proof. Again, we are going to show that every algorithm has a performance with at least $n+x-1$ steps. To every stage of the tournament we assign a natural number, $a+3b$, where a is the number of players who have either lost a game but haven't won any or have won a game and haven't lost any, and b is the number of those who have both lost and won games.

When we start, the assigned number is obviously zero. At the end we know L, which is the largest element, so all other players have lost, and we know M, the least element, so all other players have won. The assigned number is $2+3(n-2) = 3n-4$. If we can force the number to increase by at most 2 at every step, the number of steps is at least $\frac{3}{2}n-2$. If $n = 2x+2$, this is $3x+1 = n+x-1$, if $n = 2x+1$, this is $3x - \frac{1}{2}$, so the number of steps is at least $3x = n+x-1$.

If A_i and A_j are compared, A_i has won but hasn't lost, A_j has lost but hasn't won, then we make

$A_i > A_j$. Similarly, in all other cases we try to change the status of A_i, A_j as little as possible. Case analysis shows that it is possible to arrange that $a + 3b$ changes by at most 2.

For the interested reader we recommend S. Baase: Computer algorithms, Addison–Wesley, 1988 and D. E. Knuth: The art of computer programming, Vol. 3, Sorting and Searching, Addison–Wesley, 1973, as further reading.

Péter Komjáth

A Physical Experiment*

Assume that a so-called "double-doored" pocket watch is closed so it is soundproof. Is there a way to determine (without opening the door) whether or not the watch is running?

The question can be solved by a simple application of elementary principles of mechanics, more specifically, it can be decided by a simple experiment. If the watch is running, the force of the tiny spring on the swing-wheel creates a rotating/swinging motion. The 2 ends of the spring are attatched to the swing-wheel and the body of the watch, so the body of the watch is acted on by a force of equal magnitude of that on the swing-wheel in the opposite direction. If we place the watch horizontally, the body of the watch tries to move in a rotating/swinging motion around the vertical axes of the swing-wheel. The problem is that the motion is arrested by the surface friction. We therefore need to eliminate this friction, so the watch will move under the force of the spring. The easiest solution is to suspend the watch with its face horizontal. If the watch is running, it will rotate back and forth around a vertical axes; if it is not running, the watch will not move.

We can perform the experiment using the simplest materials. Following the figure, we cut out a disk and a rectangle of cardboard. We tie these together with thread (using a needle to pass the thread through the cardboard) in the same way as in the figure. We glue a small mirror to the rectangle (1–2 cm is big enough — available in any glass store for a few amount). We then suspend the entire apparatus, for example from a hanging lamp, and in a way that if it should fall, it will not fall more than a few centimeters. If we place the watch onto the plate, the entire apparatus will start to rotate quickly due to the twist of the thread, from time to time stopping and changing direction, eventually getting slower. When it slows down, we make the following observation: we look in the mirror at a distant object and observe if the movement is uniform or broken. If the watch is running, its movement interrupts the smooth slow motion of the plate. We can clearly observe this movement if we could succeed to stop the rotation of the plate, which is not an easy task as the slightest movement of air will start to slowely rotate the apparatus again. If we instead observe the apparatus when it stops to changes direction of rotation, something that typically lasts a few seconds, we can clearly see movement due to the swing-wheel that has a period of about 0.4 seconds. If the watch is not running, this movement is not present and the rotation is smooth and slow, back and forth.

Pál Selényi

* This problem was published in 1935 by the excellent Hungarian experimental physicist Pál Selényi (1884–1954)

Mathematics Problems and Solutions (already published)

F. 2967. *There are 9 points inside a unit square. Prove that among all possible triangles, there are at least 2 which have area less than 1/8.*

Solution. We first show that a triangle drawn inside a rectangle can have at most 1/2 the area of the rectangle. In *Fig. 1* we drop a perpendicular from point A to side a. The subtriangles arising on both sides of the perpendicular have an area of at most half the subrectangles containing them, so $\text{Area}_{ABC} \leq \frac{ab}{2}$. If one of the points is in the interior, then

$$\text{Area}_{ABC} < \frac{ab}{2}.$$

Fig. 1 *Fig. 2* *Fig. 3*

We divide the unit square into four rectangles of size $1 \times 1/4$ by lines parallel to two of the sides (*Fig. 2*). As there are 9 points in the square, at least one rectangle contains 3 points. None of these points can fall on the sides of size $1/4$, so these three points form a triangle of size less than $1/8$. If we perform the same division of the square with lines parallel to the other two sides, we again find a triangle of size less than $1/8$. If this triangle is different from the other, we are done.

If the two divisions give the same three points, we have *Fig. 3*. We show that for any fourth point D, there are two points among A, B, C that form a triangle with D of area less than $1/8$. We may assume that the line segment DB lies between lines segments DA and DC (it may be collinear with one of the two). We can bound the area of the quadrilateral $ABCD$ as follows: $AB < \sqrt{(1/4)^2 + (1/4)^2} = \frac{\sqrt{2}}{4}$, and $DC < \sqrt{2}$, so $\text{Area}_{ABCD} < \frac{1}{2} \cdot \frac{\sqrt{2}}{4} \cdot \sqrt{2} = \frac{1}{4}$. So either triangle DAB or triangle DBC has area less than half of that of $ABCD$, so less than $1/8$. This proves the claim.

Gy. 2409. *Thirty mailbox keys are dropped into 30 locked mailboxes randomly, one into each box. We break open one of the mailboxes. What is the probability that we will not be able to open all the remaining boxes without breaking any?*

What is the probability if we start by breaking open two mailboxes?

Solution. We claim that if we have n mailboxes and start by breaking open k (with $1 \leq k \leq n$), then the probability is $\frac{k}{n}$ that we will be able to open all n.

We start by calling this probability $p_{n,k}$.

Clearly, $p_{k,k} = 1$. Let $n > k$ and consider the n^{th} mailbox. If it contains its own key, we won't be able to open it. The chance of this is $1/n$. If the n^{th} box has the i^{th} key and the n^{th} key is in the j^{th} box $(1 < i, j < n)$ then we can consider the similar situation with $n-1$ states, with the i^{th} key in the j^{th} box. So with this observation we have

$$p_{n,k} = \left(1 - \frac{1}{n}\right) p_{n-1,k} = \left(1 - \frac{1}{n}\right)\left(1 - \frac{1}{n-1}\right) \cdots \left(1 - \frac{1}{k+1}\right) p_{k,k} =$$

$$= \frac{n-1}{n} \cdot \frac{n-2}{n-1} \cdot \ldots \cdot \frac{k}{k+1} = \frac{k}{n}.$$

Gy. 2864. *Is it possible to write the numbers* $0, 1, \ldots, 9$ *in a circle so that the difference between neighboring numbers is* $3, 4,$ *or* 5*?*

Solution. Among the numbers 0, 1, 2, 8, and 9, there are no two having a difference of 3, 4, or 5. It follows that no two of these numbers can be neighbors on the circle. Any possible solution would have these numbers lying on the circle with the remaining five numbers put in the spaces between them. As there are five spaces and five numbers remaining, each space would have to contain one number. Consider the number 7; there is no place to put it. Among the numbers 0, 1, 2, 8, 9, the number 7 only has a difference of 3, 4, or 5 with the number 2, but we must place number 7 so that it has two neighbours. This shows that such a way of arranging the numbers in a circle does not exist.

Gy. 2865. *A balance with weights in its pans sits on a classroom table. At the start, the right side is heavier. Among the weights, there are no two equal. Every weight has names of students written on it (at least one name on each). When a student comes into the classroom, the weight with his/her name (if such a weight exists) is put into the pan on the other side. Show that it is possible to send a group of students into the room, one after the other, so that at the end, the left side is heavier.*

Solution. If two ways of sending students into the room differ in order only, then the same weights will be in the same pan. The final place for a weight depends only on how many times it went from one pan to the other, that is, how many students entered the room that are listed on the weight, independent of the order.

We let s be the difference between the right and the left pans. Before starting we see that s is negative. The problem is then to show that there is a way of sending some students in so that s will be positive. We will show that the sum of all s's for sending in students in all possible ways is zero. (We assume here that students can only enter once and the order does not count.) This is clearly enough as we always had $s \leq 0$, with the starting position $s < 0$.

Every s corresponds to a sum of the weights with ± 1 coefficients, and thus the sum of the s's is a combination of the weights with integer coefficients. We show that each of these coefficients is zero.

We choose an arbitrary weight and a name that is on it. We consider all possible ways of sending the remaining students into the room. From these, we get all possible ways of sending the students in the room, by either sending our selected student in or not.

In the first case the coefficient of our selected student is $+1$ and in the second -1. These clearly add up to 0. This proves the claim.

Gy. 2867. *Hexagon $ABCDEF$ is inscribed in a circle, its* 3 *pairs of opposite sides are parallel, and the hexagon has twice the area of the triangle ACE. Show that opposite sides are equal in length.*

Solution. The hexagon is inscribed, therefore convex. Draw lines from vertices A, C, E, parallel to sides EF and BC, AB and DE, and AF and CD, respectively. Because of convexity, the pairwise intersection of the new lines are in the interior of the triangle ACE. Label these points B', D', and F' according to the Figure.

Because of the parallel lines, we have $\triangle AEF \cong AEF'$, $\triangle ABC \cong \triangle AB'C$, and $\triangle CDE \cong \triangle CD'E$. Therefore, the area of the hexagon $ABCDEF$ is

$$\begin{aligned} A &= A_{AEF} + A_{AEF'} + A_{ABC} + A_{AB'C} + A_{CDE} + A_{CD'E} + A_{B'D'F'} = \\ &= 2A_{AEF'} + 2A_{AB'C} + 2A_{CD'E} + A_{B'D'F'} = 2A_{ACE} - A_{B'D'F'}. \end{aligned}$$

According to our assumption, $A = 2A_{ACE}$, and therefore $A_{B'D'F'} = 0$, so B', D', and F' are the same point. This means $AF' = AB'$ and because of congruent triangles, $EF = AF' = AB' = BC$. Similarly, $AB = ED$ and $AF = CD$. This proves the claim.

Note. The solution does not use the fact that the hexagon is inscribed, only that it is convex.

Gy. 2868. *One of the lines connecting the centers of opposite sides of a convex quadrilateral divides its area in half and goes through the intersection point of the diagonals. Prove that the quadrilateral is a trapezoid.*

Solution. According to the figure label the four vertices of the quadrilateral A, B, C, D. Call E the midpoint of AB and F the midpoint of CD. The line EF divides the area of the quadrilateral in half. From A and B we draw perpendiculars to the line CD meeting at K and L, respectively, and from F to line AB meeting at M. We show that $AK = LB$, which proves the claim.

The line EF divides the area of the quadrilateral in half, therefore

$$A_{AEFD} = A_{AEF} + A_{AFD} = A_{BEF} + A_{BFC} = A_{BEFC}.$$

Rewriting the area of the triangles in terms of their bases and heights, we have

$$\frac{1}{2}AE \cdot FM + \frac{1}{2}FD \cdot AK = \frac{1}{2}BE \cdot FM + \frac{1}{2} \cdot FC \cdot BL.$$

We have that $AE = BE$ and $FD = FC$. From this it follows that $AK = BL$.

Note. The solution does not use the fact that the line EF goes through the intersection point of the diagonals.

Physics Problems and Solutions (already published)

Experimental problem. *In fluids and gases the motion of a body is impeded by drag force, made up of two components. One of them is the result of internal friction and it is proportional to the velocity of the body. The other force is caused by the vortices behind the body and it is proportional to the square of the velocity. Determine by measurement which force is more important in the case of a balloon falling free in the air.*

Solution. If the balloon falls from very high, we can observe that first it accelerates, then moves uniformly. When the balloon falls uniformly the sum of the forces acting on it should be zero. That means, the force of gravity is the opposite of the drag force. The constant velocity of the balloon can be determined by measuring the distance it covered and the time. The connection between the velocity and the drag force can be determined by changing the weight of the falling object. We can do that suspending small weights on the balloon.

Fig. 1 *Fig. 2*

The average velocity and drag force values obtained by measurements are plotted in *Fig. 1.* To answer the question of the problem, we should determine, whether these points are close to a first order or a second order function. To decide this we should consider the expression for the force which is the result of internal friction

$$F_f = 6\pi\eta r v, \qquad (1)$$

and for the force caused by the vortices behind the body:

$$F_v = c\varrho r^2 v^2, \tag{2}$$

(η is viscosity of the air, r is the radius of the balloon, v is its velocity, ϱ is the density of the air, c is a constant depending on the shape of the body.)

Using the SI system in expression (1) and (2) we may take their logarithm:

$$\lg F_f = \lg(rv) + \lg(6\pi\eta), \tag{3}$$
$$\lg F_v = 2\lg(rv) + \lg(c\varrho). \tag{4}$$

Plotting $\lg F_f$ and $\lg F_v$ as function of $\lg(rv)$, the lines that we obtain have a slope of 1 and 2, respectively.

If we plot the logarithm of the measured drag force as the function of $\lg rv$, only the slope of the line should be determined.

The values plotted in *Fig. 2* were obtained by the student Krisztián Tar. The points are close to a line with slope 2. So we can say that the force caused by the vortices behind the body is more important in the case of a falling balloon.

Theoretical problems

2718. *In which case is a smaller force required to hold the same bucket: when it is filled with air in the air or filled with water under water?*

Küldte: *Competition problem, Hungary*

Solution. We do not need to exert any force to hold the matter in the bucket, as the resultant of the forces acting on it (gravitational force and upthrust) is zero. The gravitational force and the upthrust act also on the bucket in both cases. We have to exert the force of reaction against the resultant of these forces to hold the bucket:

$$F = |mg - \varrho_{\mathrm{s}} V g| = |\varrho_{\mathrm{bu}} - \varrho_{\mathrm{s}}| V g,$$

where $m = \varrho_{\mathrm{bu}} V$ is the mass of the bucket, V is the volume of the material of the bucket and ϱ_{s} is the density of the given substance.

If $\varrho_{\mathrm{bu}} > \varrho_{\mathrm{water}}$ then it is obviously easier to hold the bucket in water. This is also true if $\varrho_{\mathrm{bu}} < \varrho_{\mathrm{water}}$ but

$$\varrho_{\mathrm{water}} - \varrho_{\mathrm{bu}} < \varrho_{\mathrm{bu}} - \varrho_{\mathrm{air}},$$

that is

$$\varrho_{\mathrm{bu}} > \frac{\varrho_{\mathrm{water}} + \varrho_{\mathrm{air}}}{2}.$$

This condition is generally fulfilled. If ϱ_{bu} is smaller than the arithmetic mean of the densities of water and air then it is easier to hold it in air. If $\varrho_{\mathrm{bu}} < \varrho_{\mathrm{water}}$ then "holding" means stopping the bucket from rising.

2781. *At what angle shall we throw a stone if we want it to travel away from us at every moment? (The air resistance can be neglected.)*

Küldte: *Russian competition problem*

Solution. The position and the speed of a stone thrown at angle α and with initial speed v_0 are given by the relations

$$x = v_0 t\cos\alpha, \qquad v_x = v_0\cos\alpha, \qquad y = v_0 t\sin\alpha - \frac{g}{2}t^2, \qquad v_y = v_0\sin\alpha - gt.$$

A stone thrown at a sufficiently large angle will first get further away from its initial position and then closer again. At the moment when the stone is the furthest away from us its velocity is perpendicular to its position vector. Then:

$$\frac{x}{y} = \frac{-v_y}{v_x}, \qquad \text{so} \qquad xv_x + yv_y = 0.$$

Using the above formulae we get the following second degree equation for the time t corresponding to the furthest position of the stone:

$$\frac{g^2}{2}t^2 - \frac{3}{2}gv_0 \sin\alpha \cdot t + v_0^2 = 0.$$

If the discriminant of this equation is negative, *there is no time* when the body is at its "furthest position" from us, i.e. the stone keeps increasing its distance from us. This happens if

$$\sin^2\alpha < \frac{8}{9}, \qquad \text{so} \qquad \alpha < 70.5°.$$

Note. The problem can also be considered as a problem in coordinate geometry concerning the curve of the path: when does the function $F(x) = x^2 + y(x)^2$ not have a maximum if $y = ax - bx^2$? Differentiating and examining the discriminant of a second degree equation we get the condition $a < \sqrt{8}$ for the constant increase in the distance; using $a = \operatorname{tg}\alpha$ this is equivalent to the result obtained on the basis of physical considerations.

2553. *One arm of a U-shaped, vertically placed tube of uniform cross-section is closed, the other, longer one is open. There is mercury in the tube as shown in the picture.*

What is the length of the column of air enclosed in the tube if we find that its pressure is directly proportional to its volume when heated? The external pressure is equal to the pressure of a 76 cm *high column of mercury.*

Küldte: *Submitted by Gyula Radnai,* Budapest

Solution. Let us denote the length of the column of air enclosed in the tube by h, the cross-section of the tube by A, and let us measure the pressures in Hgcm and the lenghts in cm units. Initially the pressure of the gas is $p_1 = 76 + h$ and its volume is $V_1 = hA$.

Fig. 1

If the column of mercury is displaced by x due to the heating, the pressure of the gas will be $p_2 = 76 + h + 2x$, and its volume $V_2 = (h + x)A$. According to the given condition

$$\frac{p_1}{V_1} = \frac{p_2}{V_2},$$

that is

$$\frac{76 + h}{h} \equiv \frac{76 + h + 2x}{h + x}.$$

This relation is fulfilled for arbitrary x if $h = 76$, so the length of the column of air enclosed in the tube is 76 cm.

2230. *We want to produce pressure p in a vessel of volume V_2 using the air compressor shown in the figure. The piston can move without friction in a cylinder of volume V_p and constant cross-section. At the beginning the air in the vessel has the same pressure and temperature as the surrounding atmosphere. Calculate the mechanical work necessary to produce pressure p in the vessel. (Assume that the processes are isothermal and there is no work done while pulling the piston.)*

Solution. Let there be N molecules in the vessel of volume V_2 at the end of the process. As they form a gas of pressure p_0 at the beginning and p at the end, their *entropy* has changed by the value

$$\Delta S = N \cdot k \ln \frac{p_0}{p}$$

in the course of the process. As $p > p_0$ the entropy of the gas has decreased, and the system has come to a more ordered state.

On the other hand it is true that $\Delta S = \frac{Q}{T} + \sigma$, where Q is the heat absorbed by the system (at constant temperature T) and σ is the contribution of *irreversible processes.* In our case $\sigma = 0$ and $Q < 0$ as the compressed gas gives out heat to its surroundings.

As we have assumed that the temperature and so the internal energy of the gas remains constant during the process

$$\Delta E = Q + W = 0,$$

thus

$$W = -Q = -T \cdot \Delta S = -NkT \cdot \ln \frac{p_0}{p} = p \cdot V_2 \cdot \ln \frac{p}{p_0}.$$

Note. W as calculated above is not identical to the energy necessary to compress the gas, but is bigger. The difference is due to the fact that a certain proportion of the work is "done" by the external atmospheric pressure; the person using the air compressor only has to do

$$W' = W - (p - p_0)V_2 = \left[\ln \frac{p}{p_0} - \left(1 - \frac{p_0}{p}\right)\right]$$

work. If $p \gg p_0$ then this difference is negligible, but if we do not aim for a very large ratio of compression then it is significant.

2565. *In the circuit shown in the diagram the terminal voltage between points A and B is* 300 V *and the resistors are of* 100 Ω *each. What do the meters denoted by M show if they are*

a) *ideal voltmeters,*

b) *ideal ammeters?*

Küldte: *Submitted by István Varga,* Békéscsaba

Solution.

Fig. 1

a) If the meters are ideal voltmeters ($R_i = \infty$) then they show the p.d. on a resistance of $R_1 = R + R =$ 200 Ω. As the values of the three resistances are identical the p.d. $V_{AB} = 300V$ will be divided equally among them, so both voltmeters will show 200 V (*Fig. 1*). The same can be derived from Ohm's law as follows.

$$I = \frac{V_{AB}}{3R} = \frac{300\text{ V}}{300\ \Omega} = 1\text{ A}, \qquad V_{CB} = I \cdot 2R = 200V.$$

Fig. 2

b) If the meters are ideal ammeters ($R_i = 0$) then we can redraw the circuit diagram in a better way (*Fig. 2*). In this case there are three resistors in parallel and an ammeter measures the current flowing through two resistors. The current taken from the voltage step-up is $I = U_{AB}/R_r$, where

$$\frac{1}{R_r} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}, \qquad R_r = 33.3\ \Omega.$$

So a total current of $I = 300V/33.33\Omega = 9A$ flows through the three resistors. The current flowing through each resistor is

$$I_R = \frac{I}{3} = 3\text{ A}.$$

So both ammeters show a current of $2I_R = 6A$.

2733. *Assume that the temperature of the filament of a tungsten filament lamp is directly proportional to the absolute temperature, and the energy radiated from its surface is proportional to the fourth power of the absolute temperature. What is the relation between the current flowing through the filament and the voltage connected to it?*

Küldte: *English competition problem*

Solution. From the equality of the powers taken from the mains and radiated

$$U \cdot I \sim T^4.$$

But as $R = \frac{U}{I} \propto T$ we can write $U \cdot I \sim \left(\frac{U}{I}\right)^4$, so $I^5 \propto U^3$ or in a different way

$$I(U) = \text{constant} \cdot U^{\frac{3}{5}}.$$

2625. *Today the ratio of the isotopes* ^{235}U *to* ^{238}U *in natural uranium is* 1 : 139. *The half-life of* ^{238}U *is* $4.5 \cdot 10^9$ *years, that of* ^{235}U *is* $7.1 \cdot 10^8$ *years. What percents of the initial* ^{238}U *nuclei are left after this latter time has elapsed?*

On the basis of the above data, give an estimate of the time of the supernova explosion in which the heavy elements of the solar system were formed. (According to our knowledge of nuclear physics on Earth, there were not more ^{235}U *than* ^{238}U *formed in the explosion.)*

Küldte: *Submitted by László Madas,* Budapest

Solution. Let T_{235} and T_{238} denote the half-lives. After the given time $t = T_{235}$ the number of undecayed ^{238}U nuclei is

$$N(t) = N_0 \cdot 2^{-t/T_{238}},$$

so $2^{-T_{235}/T_{238}} \approx 89.6\%$ of the initial nuclei still remain. At the time of the supernova explosion in question, $N_{235}(0) \le N_{238}(0)$, and today $\frac{N_{235}(t)}{N_{238}(t)} = \frac{1}{139}$. Dividing the two laws of decay we get

$$\frac{N_{235}(t)}{N_{238}(t)} = \frac{N_{235}(0) \cdot 2^{-t/T_{235}}}{N_{238}(0) \cdot 2^{-t/T_{238}}} \le 2^{t(1/T_{238} - 1/T_{235})}.$$

From this $t \le \frac{\log_2(1/139)}{1/T_{238} - 1/T_{235}} \approx 6 \cdot 10^9$ years.

Therefore the supernova explosion could happen at most 6 thousand million years ago; so in the mean time we have also given an estimate for the upper limit of the age of the solar system.

A selection of KöMal problems from 1993–94

Here are some examples of exercises from issues of the journal that appeared in the 1993/94 school year. We have included problems of all types, in the same number that usually appears in one issue. The students are generally given one month to solve, write up, and send in solutions to the problems. At the end of every school year, we list the final comprehensive results.

Problems are assembled by the mathematical and physical editorial boards.

Mathematics

C exercises (Easier exercises)

C. 341. The population of the Earth will reach 6 billion by the year 2000. Fewer than 1% will reach the age of 100. Prove that there will be 100 people born in the same minute.

C. 358. Prove that the sum of the fourth powers of three whole numbers is divisible by 7 if the sum of their squares is divisible by 7.

C. 339. We have to drill three holes in a circular disk so that the maximal distance between any two holes and between a hole and the edge of the disk be maximal. Determine where to drill the holes.

C. 364. An Aztec pyramid (rectangular) is truncated at the top so that the edge dimensions of the top are 16m long. The width of the pyramid is 81m and the depth is 65 m. In the interest of tourists, stairs are built, starting from a ground corner, covering all four sides, and rising with constant steepness. At what points does the stairway intersect the sides?

Exercises (for 14–16 year-old students)

Gy. 2910. Is it possible to write the numbers 111, 112, 121, 122, 211, 212, 221 and 222 on the vertices of a cube such that the numbers written on two neighbouring vertices differ in at least two digits?

Gy. 2857. In a school, each of the languages English, German and French is spoken by exactly 50 students. (There may be students who speak more than one languages and also who do not speak any of these languages.) Prove that the students can be divided into 5 groups so that each language is spoken by exactly 10 students in each group.

Gy. 2888. There are a few islands in a 100 m wide river. Their perimeters add up to 800 m. The banks of the river are parallel lines. Gabriel states that wherever he starts, he is able to cross the river by rowing at most 300 m. May one believe this?

Gy. 2879. The product of the positive divisors of a natural number is equal to 8000. Find all natural numbers that have this property.

Gy. 2877. It is well known that for every triangle, its smallest angle is incident to its longest side. Is the same statement true for quadrilaterals as well?

Gy. 2885. Consider the lines determined by the edges of a regular tetrahedron. Rotate each line about the opposite line through an angle of 180°. What kind of a configuration do the six new lines form?

Gy. 2925. The billstickers of the Mathematicians' Party observed that people read the posters standing 3 meters away from the centres of the advertising pillars that have 1.5 m diameter. The Party wants to arrange things so that after putting posters all around a pillar, a whole poster would be visible from any direction. How wide should the posters be?

Küldte: *Submitted by P. Hanyecz,* Budapest

Gy. 2901. Reflect the unit cube about its faces. Consider the body consisting of the original cube and its six images. Reflect this new polytope again about the plane of each of its faces. Find the volume of the body obtained this way.

Problems (for 17–18 year-old students)

F. 2975. The product of the positive numbers a, b and c is equal to 1. Suppose furthermore that the inequality

$$a + b + c > 1/a + 1/b + 1/c$$

holds. Prove that exactly one of the numbers a, b and c is greater than 1.

F. 2976. A die is thrown — at most one hundred times — until the sum of some consecutive scores is either 6 or 7. Which event is more likely?

Küldte: *Submitted by L. Pap,* Szeged

F. 2986. Find those positive integers n for which there exist positive integers a and b such that $ab(a+b)$ and n are coprime numbers. How does the solution change if we add the extra condition that $a - b$ should not be divisible by n?

Küldte: *Submitted by Ervin Fried,* Budapest

F. 2973. Reflecting the orthocentre of a triangle in the lines of its sides, we obtain a convex hexagon having angles $130°, 140°, 110°, 80°, 120°$ and $140°$, in that order. The area of the hexagon is more than twice as large as the area of the original triangle. Find the greatest angle of the triangle.

Küldte: *Submitted by G. Bakonyi,* Budapest

F. 2991. We spread out the triangular faces of a quadrilateral based pyramid on to the plane of its base. Somebody erased the images of two neighbouring triangular faces from the network of the pyramid, leaving the base edges unaltered. Construct the missing part of the network.

F. 3008. An acute-angled triangle has sides a, b, c and area T. Prove the inequality

$$\frac{T}{\sqrt{a^2b^2 - 4T^2}} + \frac{T}{\sqrt{b^2c^2 - 4T^2}} + \frac{T}{\sqrt{c^2a^2 - 4T^2}} \geq \frac{3\sqrt{3}}{2}.$$

Küldte: *Submitted by T. Árokszállási,* Paks

Advanced problems (which help to prepare for international competitions)

N. 25. Find those obtuse angles γ for which every triangle that has an angle γ satisfies inequality

$$\frac{T}{\sqrt{a^2b^2 - 4T^2}} + \frac{T}{\sqrt{b^2c^2 - 4T^2}} + \frac{T}{\sqrt{c^2a^2 - 4T^2}} \geq \frac{3\sqrt{3}}{2},$$

where a, b, c and T denote the sides and the area of the triangle, respectively.

Küldte: *Submitted by T. Árokszállási,* Paks

N. 21. Show that every positive integer k has an integer multiple in the interval $[1; k^4]$ the decimal expression of which contains at most four different digits.

Küldte: *Advanced problem proposed for the XXXIst International Mathematical Olympiad)*

N. 28. Find those integers n for which a regular hexagon can be divided into n parallelograms that have equal areas.

N. 20. There are n^2 bulbs attached to an $n \times n$ board. Some of them are lit. There is a switch associated with each row and column of the board. Turning a switch to its other position, changes the lights of the bulbs in the appropriate row or column to their opposite. Show that with a suitable sequence of switchings, one can achieve a state such that the difference between the number of the lit bulbs and the number of the dark bulbs is at least $\sqrt{n^3/2}$.

Physics

Experimental problem 157. Measurement of the exposition time of a camera. The luminosity of our light sources fed from the mains alternating voltage fluctuates with a frequency of 100 Hz. Make photographs of a light source with different exposition times in such a way that you turn the camera round to the side during the time of the exposition. It is expedient to choose a light tube as your light source. Send in the copy of the photograph made with an exposition of 1/25 or 1/30 seconds and the actual exposition time determined according to this photograph.

Küldte: *Adopted from Á. Csekő* (1902–1993)

Exercises (for 13–16 year-old students)

FGY. 2751. While returning a ping-pong ball arriving at a speed of 12 m/s, a player moves the paddle towards the ball at 3 m/s. What is the speed of the bouncing ball a) relative to the paddle; b) relative to the table?

Küldte: *Submitted by Gy. Radnai,* Budapest

FGY. 2803. Three points at rest (A, B, C) are placed along a level straight line in such a way that $AB = BC$. All the three points start moving at the same time: point A with a constant velocity $v_0 = 10$ m/s vertically upwards, point C with no initial speed and an acceleration of $a = 2$ m/s^2 vertically downwards, and point B in such a way that the three points always be on the same straight line. How high will point B rise?

Küldte: *Submitted by L. Kotek,* Pécs

Problems (for 17–18 year-old students)

FF. 2763. A flea tries to improve its individual record in the long jump. It puts its little brother on its back and jumps, doing the same amount of work as if it jumped alone. At the highest point of its trajectory it thrusts its brother horizontally backwards. The work done is the same as that for jumping. Can it be successful in improving its record?

Küldte: *Submitted by I. Varga,* Békéscsaba

FF. 2756. Somewhat away from the edge of a horizontal table, there is a cylinder-shaped spool of yarn. The yarn is fastened to a heavy object thrown over a pulley. Then the system is left alone. Is it faster for the spool to get to the edge of the table on a rough or on a smooth surface?

Küldte: *Submitted by P. Gnädig,* Budapest

FF. 2782. We make a tetrahedron from resistors that look similar. The resistances of all but one are 2 kΩ. We make measurements with an ohmmeter, i.e. we can determine the resulting resistance between any two tips of the tetrahedron.

a) How many types of resistances can we measure?

b) What are these resistances if the unknown resistance is r?

c) How many measurements shall we at least make to determine which is the unknown resistor and how big is r?

Küldte: *Chinese problem*

FF. 2783. A long coil (solenoid) contains another solenoid (whose radius is half of its own) coaxially. The number of turns on a unit lenght of both coils is the same. Smoothly increasing the current flowing in the coils from zero, an electric field is induced in the surroundings of the coils. The current flowing in the inner coil is twice as large as that in the outer one at any moment and their directions are the same. A charged particle initially stationary in the space between the two coils moves on a circular path due to the electromagnetic forces. What is the radius of the circle?

Küldte: *Submitted by F. Schuszter,* Budapest

FF. 2807. In a direct current circuit a copper wire of cross sectional area 10 mm^2 is continued by an iron wire of cross sectional area 10 mm^2. A current of $I = 10$ A flows in the direction marked in the diagram. How much charge of what signature is accumulated in the contacting cross-sections of the two wires? How many elementary charges is this equivalent to?

Küldte: *Submitted by L. Holics,* Budapest

Advanced problem (which help to prepare for international competitions)

FN. 2784. There is a circular hole of radius r on the bottom of a container with a ball of mass m and radius R on it. If there is enough water in the container the ball stays on the bottom. Cautiously decreasing the quantity of the water the ball rises at a certain water height h_0. Find out the value of h_0.

Küldte: *Submitted by T. Bíró,* Marosvásárhely