

The pentagon  $ABCDE$  has an inscribed circle of centre  $O$  and radius  $r$ . Given that the angle at vertex  $A$  is a right angle,  $\angle EOA = 60^\circ$ , and that triangle  $OCD$  is equilateral, find the area of the pentagon.