I. Megoldás.

$$1-\sin\frac{\alpha}{2}=1-\cos\left(90^\circ-\frac{\alpha}{2}\right)=2\sin^2\left(45^\circ-\frac{\alpha}{4}\right)=$$
$$=2\left(\sin 45^\circ\cos\frac{\alpha}{4}-\cos 45^\circ\sin\frac{\alpha}{4}\right)^2=2\left(\frac{\sqrt{2}}{2}\right)^2\left(\cos\frac{\alpha}{4}-\sin\frac{\alpha}{4}\right)^2=$$
$$=\left(\frac{\alpha}{4}-\sin\frac{\alpha}{4}\right)^2.$$

Továbbá $$\cos\frac{\alpha}{2}=\cos^2\frac{\alpha}{4}-\sin^2\frac{\alpha}{4}.$$

Ezek alapján

$$\frac{1-\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}=\frac{\left(\cos\frac{\alpha}{4}-\sin\frac{\alpha}{4}\right)^2}{\cos^2\frac{\alpha}{4}-\sin^2\frac{\alpha}{4}}=\frac{\cos\frac{\alpha}{4}-\sin\frac{\alpha}{4}}{\cos\frac{\alpha}{4}+\sin\frac{\alpha}{4}}=$$
$$=\frac{1-\operatorname{tg}\frac{\alpha}{4}}{1+\operatorname{tg}\frac{\alpha}{4}}=\operatorname{tg}\left(45^\circ-\frac{\alpha}{4}\right)=\operatorname{cotg}\left(45^\circ+\frac{\alpha}{4}\right).$$

Bizám György (Bolyai g. VI. o. Bp. V.).

II. Megoldás

$$\frac{1-\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}=\frac{1-\sin\frac{\alpha}{2}}{\sqrt{1-\sin^2\frac{\alpha}{2}}}=\sqrt{\frac{1-\sin\frac{\alpha}{2}}{1+\sin\frac{\alpha}{2}}}=$$
$$=\sqrt{\frac{1-\cos\left(90^\circ-\frac{\alpha}{2}\right)}{1+\cos\left(90^\circ-\frac{\alpha}{2}\right)}}=\sqrt{\frac{2\sin^2\left(45^\circ-\frac{\alpha}{4}\right)}{2\cos^2\left(45^\circ-\frac{\alpha}{4}\right)}}=\operatorname{tg}\left(45^\circ-\frac{\alpha}{4}\right).$$

Hibbey Levente (Fáy András g. VII. o. Bp. IX.).