

I. Megoldás.

$$\begin{aligned} 1 - \sin \frac{\alpha}{2} &= 1 - \cos \left(90^\circ - \frac{\alpha}{2} \right) = 2 \sin^2 \left(45^\circ - \frac{\alpha}{4} \right) = \\ &= 2 \left(\sin 45^\circ \cos \frac{\alpha}{4} - \cos 45^\circ \sin \frac{\alpha}{4} \right)^2 = 2 \left(\frac{\sqrt{2}}{2} \right)^2 \left(\cos \frac{\alpha}{4} - \sin \frac{\alpha}{4} \right)^2 = \\ &= \left(\cos \frac{\alpha}{4} - \sin \frac{\alpha}{4} \right)^2. \end{aligned}$$

Továbbá
Ezek alapján

$$\cos \frac{\alpha}{2} = \cos^2 \frac{\alpha}{4} - \sin^2 \frac{\alpha}{4}.$$

$$\begin{aligned} \frac{1 - \sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} &= \frac{\left(\cos \frac{\alpha}{4} - \sin \frac{\alpha}{4} \right)^2}{\cos^2 \frac{\alpha}{4} - \sin^2 \frac{\alpha}{4}} = \frac{\cos \frac{\alpha}{4} - \sin \frac{\alpha}{4}}{\cos \frac{\alpha}{4} + \sin \frac{\alpha}{4}} = \\ &= \frac{1 - \operatorname{tg} \frac{\alpha}{4}}{1 + \operatorname{tg} \frac{\alpha}{4}} = \operatorname{tg} \left(45^\circ - \frac{\alpha}{4} \right) = \operatorname{cotg} \left(45^\circ + \frac{\alpha}{4} \right). \end{aligned}$$

Bizám György (Bolyai g. VI. o. Bp. V.).

II. Megoldás

$$\begin{aligned} \frac{1 - \sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} &= \frac{1 - \sin \frac{\alpha}{2}}{\sqrt{1 - \sin^2 \frac{\alpha}{2}}} = \sqrt{\frac{1 - \sin \frac{\alpha}{2}}{1 + \sin \frac{\alpha}{2}}} = \\ &= \sqrt{\frac{1 - \cos \left(90^\circ - \frac{\alpha}{2} \right)}{1 + \cos \left(90^\circ - \frac{\alpha}{2} \right)}} = \sqrt{\frac{2 \sin^2 \left(45^\circ - \frac{\alpha}{4} \right)}{2 \cos^2 \left(45^\circ - \frac{\alpha}{4} \right)}} = \operatorname{tg} \left(45^\circ - \frac{\alpha}{4} \right). \end{aligned}$$

Hibbey Levente (Fáy András g. VII. o. Bp. IX.).