

(a)

$$(x+y)^3 - x^3 - y^3 = x^3 + 3x^2y + 3xy^2 + y^3 - x^3 - y^3 = 3x^2y + 3xy^2 = 3xy(x+y)$$

(b)

$$\begin{aligned}(x+y)^5 - x^5 - y^5 &= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5 - x^5 - y^5 = \\ &= 5x^2y(x^2 + xy + y^2) + 5xy^2(x^2 + xy + y^2) = 5xy(x+y)(x^2 + xy + y^2)\end{aligned}$$

(c)

$$\begin{aligned}(x+y)^7 - x^7 - y^7 &= x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + \\ &+ 21x^2y^5 + 7xy^6 + y^7 - x^7 - y^7 = 7x^6y + 14x^5y^2 + \\ &+ 7x^4y^3 + 14x^4y^3 + 14x^3y^4 + 7x^2y^5 + 7x^5y^2 + \\ &+ 14x^4y^3 + 7x^3y^4 + 14x^3y^4 + 14x^2y^5 + 7xy^6 = \\ &= 7x^2y(x^4 + 2x^3y + x^2y^2 + 2x^2y^2 + 2xy^3 + y^4) + \\ &+ 7xy^2(x^4 + 2x^3y + x^2y^2 + 2x^2y^2 + 2xy^3 + y^4) = \\ &= (7x^2y + 7xy^2)(x^2 + 2xy + y^2)^2 = \\ &= 7xy(x+y)(x^2 + xy + y^2)^2.\end{aligned}$$

(Lówy József, Losoncz.)

Megoldások száma: 32.