1°. Ismeretes, hogy *(Mathematikai Gyakorlókönyv II. 360. f.)*

$$\text{tg}\ \alpha + \text{tg}\ \beta + \text{tg}\gamma = \text{tg}\ \alpha\ \text{tg}\ \beta\ \text{tg}\gamma. \tag{1}$$

Ha eme egyenlet mindkét oldalát négyzetre emeljük, ered

$$A = \text{tg}^2\ \alpha\ \text{tg}^2\ \beta\ \text{tg}^2\gamma - (\text{tg}^2\ \alpha + \text{tg}^2\ \beta + \text{tg}^2\ \gamma) =$$

$$= 2(\text{tg}\ \alpha \text{tg}\ \beta + \text{tg}\alpha\ \text{tg}\gamma + \text{tg}\ \beta\ \text{tg}\gamma) =$$

$$= 2\sec\alpha\sec\beta\sec\gamma(\sin\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma\cos\beta + \sin\alpha\sin\gamma\cos\alpha) =$$

$$= 2\sec\alpha\sec\beta\sec\gamma(\sin\alpha\sin\beta\cos\gamma + \sin^2\gamma) =$$

$$= 2\sec\alpha\sec\beta\sec\gamma[1 + \cos\gamma(\sin\alpha\sin\beta - \cos\gamma)] =$$

$$= 2\sec\alpha\sec\beta\sec\gamma[1 + \cos\alpha\cos\beta\cos\gamma)] = 2 + 2\sec\alpha\sec\beta\sec\gamma.$$

2°. Ha (1)-et köbre emeljük, ered

$$B = \text{tg}^3\ \alpha\ \text{tg}^3\ \beta\ \text{tg}^3\gamma - (\text{tg}^3\ \alpha + \text{tg}^3\ \beta + \text{tg}^3\gamma) =$$

$$= 3[\text{tg}^2\ \alpha\ \text{tg}\ \beta + \text{tg}\ \alpha\ \text{tg}^2\ \beta + \text{tg}^2\alpha\ \text{tg}\gamma + \text{tg}\ \alpha\ \text{tg}^2\ \gamma + \text{tg}^2\ \beta\ \text{tg}\ \gamma + \text{tg}\ \beta\ \text{tg}^2\ \gamma +$$

$$+ 3\text{tg}\ \alpha\ \text{tg}\ \beta\ \text{tg}\ \gamma - \text{tg}\ \alpha\ \text{tg}\ \beta\ \text{tg}\ \gamma] =$$

$$= 3[\text{tg}\ \alpha\ \text{tg}\ \beta\ (\text{tg}\ \alpha + \text{tg}\ \beta + \text{tg}\ \gamma) + \text{tg}\ \alpha\ \text{tg}\ \gamma(\text{tg}\ \alpha + \text{tg}\ \beta + \text{tg}\ \gamma) +$$

$$+ \text{tg}\ \beta \text{tg}\gamma)(\text{tg}\ \alpha + \text{tg}\ \beta + \text{tg}\ \gamma) - \text{tg}\ \alpha\ \text{tg}\ \beta \text{tg}\ \gamma] =$$

$$= 3\text{tg}\ \alpha\ \text{tg}\ \beta\ \text{tg}\ \gamma[\text{tg}\ \alpha\ \text{tg}\ \beta + tg\ \alpha\ \ tg\ \gamma + \text{tg}\ \beta\ \text{tg}\ \gamma - 1].$$

De, mint 1°-ben láttuk, a szögletes zárójelben álló kifejezés:

$$\sec\alpha\sec\beta\sec\gamma + 1,$$

s így

$$B = 3\ tg\ \alpha\ \text{tg}\ \beta\ \text{tg}\ \gamma\ [\sec\ \alpha\ \sec\ \beta\ \sec\gamma + 1 - 1] = \frac{3\ tg\ \alpha\ \ tg\ \beta\ \ tg\ \gamma}{\cos\ \alpha\ \cos\beta\ \cos\gamma}.$$

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