

$$\begin{aligned}
4\text{ctg}\phi &= \text{ctg}\frac{\gamma}{2} \left[\text{ctg}\frac{\alpha}{2} \text{ctg}\frac{\beta}{2} + \text{tg}\frac{\alpha}{2} \text{tg}\frac{\beta}{2} \right] + \\
&+ \text{tg}\frac{\gamma}{2} \left[\text{ctg}\frac{\alpha}{2} \text{tg}\frac{\beta}{2} + \text{tg}\frac{\alpha}{2} \text{ctg}\frac{\beta}{2} \right] = \\
&= \text{ctg}\frac{\gamma}{2} \left[\frac{\cos^2 \frac{\alpha}{2} \cos^2 \frac{\beta}{2} + \sin^2 \frac{\alpha}{2} \sin^2 \frac{\beta}{2}}{\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \sin \frac{\beta}{2} \cos \frac{\beta}{2}} \right] + \\
&+ \text{tg}\frac{\gamma}{2} \left[\frac{\cos^2 \frac{\alpha}{2} \sin^2 \frac{\beta}{2} + \sin^2 \frac{\alpha}{2} \cos^2 \frac{\beta}{2}}{\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \sin \frac{\beta}{2} \cos \frac{\beta}{2}} \right],
\end{aligned}$$

vagy

$$\begin{aligned}
\text{ctg}\phi &= \frac{\text{ctg}\frac{\gamma}{2} \left[\cos^2 \left(\frac{\alpha}{2} + \frac{\beta}{2} \right) + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \sin \frac{\beta}{2} \cos \frac{\beta}{2} \right]}{\sin \alpha \sin \beta} + \\
&+ \frac{\text{tg}\frac{\gamma}{2} \left[\sin^2 \left(\frac{\alpha}{2} + \frac{\beta}{2} \right) - 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \sin \frac{\beta}{2} \cos \frac{\beta}{2} \right]}{\sin \alpha \sin \beta} \\
&= \frac{\text{ctg}\frac{\gamma}{2} \sin^2 \frac{\gamma}{2} + \text{tg}\frac{\gamma}{2} \cos^2 \frac{\gamma}{2}}{\sin \alpha \sin \beta} + \\
&+ \frac{2 \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \cos \frac{\alpha}{2} \cos \frac{\beta}{2} \left(\text{ctg}\frac{\gamma}{2} - \text{tg}\frac{\gamma}{2} \right)}{\sin \alpha \sin \beta} = \\
&= \frac{\sin \gamma}{\sin \alpha \sin \beta} + \frac{\text{ctg}\frac{\gamma}{2} - \text{tg}\frac{\gamma}{2}}{2}.
\end{aligned}$$

De

$$\frac{\text{ctg}\frac{\gamma}{2} - \text{tg}\frac{\gamma}{2}}{2} = \frac{1 - \text{tg}^2 \frac{\gamma}{2}}{2 \text{tg}\frac{\gamma}{2}} = \text{ctg}\gamma,$$

tehát

$$\begin{aligned}
\text{ctg}\phi &= \frac{\sin \gamma}{\sin \alpha \sin \beta} + \text{ctg}\gamma = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha \sin \beta} + \text{ctg}\gamma = \\
&= \text{ctg}\alpha + \text{ctg}\beta + \text{ctg}\gamma,
\end{aligned}$$

vagyis

$$\begin{aligned}
4(\text{ctg}\alpha + \text{ctg}\beta + \text{ctg}\gamma) &= \text{ctg}\frac{\alpha}{2} \text{ctg}\frac{\beta}{2} \text{ctg}\frac{\gamma}{2} + \text{ctg}\frac{\alpha}{2} \text{tg}\frac{\beta}{2} \text{tg}\frac{\gamma}{2} + \\
&+ \text{ctg}\frac{\beta}{2} \text{tg}\frac{\alpha}{2} \text{tg}\frac{\gamma}{2} + \text{ctg}\frac{\gamma}{2} \text{tg}\frac{\alpha}{2} \text{tg}\frac{\beta}{2}.
\end{aligned}$$

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