$$4\text{ctg}\phi = \text{ctg}\frac{\gamma}{2}\left[\text{ctg}\frac{\alpha}{2}\text{ctg}\frac{\beta}{2} + \text{tg}\frac{\alpha}{2}\text{tg}\frac{\beta}{2}\right] +$$

$$+\text{tg}\frac{\gamma}{2}\left[\text{ctg}\frac{\alpha}{2}\text{tg}\frac{\beta}{2} + \text{tg}\frac{\alpha}{2}\text{ctg}\frac{\beta}{2}\right] =$$

$$= \text{ctg}\frac{\gamma}{2}\left[\frac{\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2} + \sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}}{\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\frac{\beta}{2}}\right] +$$

$$+\text{tg}\frac{\gamma}{2}\left[\frac{\cos^2\frac{\alpha}{2}\sin^2\frac{\beta}{2} + \sin^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}}{\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\frac{\beta}{2}}\right],$$

vagy

$$\text{ctg}\phi = \frac{\text{ctg}\frac{\gamma}{2}\left[\cos^2\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\frac{\beta}{2}\right]}{\sin\alpha\sin\beta} +$$

$$+\frac{\text{tg}\frac{\gamma}{2}\left[\sin^2\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) - 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\frac{\beta}{2}\right]}{\sin\alpha\sin\beta}$$

$$= \frac{\text{ctg}\frac{\gamma}{2}\sin^2\frac{\gamma}{2} + \text{tg}\frac{\gamma}{2}\cos^2\frac{\gamma}{2}}{\sin\alpha\sin\beta} +$$

$$+\frac{2\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\left(\text{ctg}\frac{\gamma}{2} - \text{tg}\frac{\gamma}{2}\right)}{\sin\alpha\sin\beta} =$$

$$= \frac{\sin\gamma}{\sin\alpha\sin\beta} + \frac{\text{ctg}\frac{\gamma}{2} - \text{tg}\frac{\gamma}{2}}{2}.$$

De

$$\frac{\text{ctg}\frac{\gamma}{2} - \text{tg}\frac{\gamma}{2}}{2} = \frac{1 - \text{tg}^2\frac{\gamma}{2}}{2\text{tg}\frac{\gamma}{2}} = \text{ctg}\gamma,$$

tehát

$$\text{ctg}\phi = \frac{\sin\gamma}{\sin\alpha\sin\beta} + \text{ctg}\gamma = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\sin\beta} + \text{ctg}\gamma =$$

$$= \text{ctg}\alpha + \text{ctg}\beta + \text{ctg}\gamma,$$

vagyis

$$4(\text{ctg}\alpha + \text{ctg}\beta + \text{ctg}\gamma) = \text{ctg}\frac{\alpha}{2}\text{ctg}\frac{\beta}{2}\text{ctg}\frac{\gamma}{2} + \text{ctg}\frac{\alpha}{2}\text{tg}\frac{\beta}{2}\text{tg}\frac{\gamma}{2} +$$

$$+\text{ctg}\frac{\beta}{2}\text{tg}\frac{\alpha}{2}\text{tg}\frac{\gamma}{2} + \text{ctg}\frac{\gamma}{2}\text{tg}\frac{\alpha}{2}\text{tg}\frac{\beta}{2}.$$

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