

I.

$$\begin{aligned}\cos^2 A + \cos^2 B + \cos^2 C &= \\&= \cos A \cos[180^\circ - (B + C)] + \cos B \cos[180^\circ - (A + C)] + \cos^2 C = \\&= -\cos A \cos(B + C) - \cos B \cos(A + C) + \cos^2 C = \\&= -2 \cos A \cos B \cos C + \sin A \sin B \sin C + \cos B \sin A \sin C + \cos^2 C = \\&= \sin C (\sin A \cos B + \cos A \sin B) + \cos^2 C - 2 \cos A \cos B \cos C = \\&= \sin C \sin(A + B) + \cos^2 C - 2 \cos A \cos B \cos C = \\&= \sin^2 C + \cos^2 C - 2 \cos A \cos B \cos C = \\&= 1 - 2 \cos A \cos B \cos C.\end{aligned}$$

II.

$$\begin{aligned}\sin^2 A + \sin^2 B + \sin^2 C &= \\&= 1 - \cos^2 A + 1 - \cos^2 B + 1 - \cos^2 C = \\&= 3 - (\cos^2 A + \cos^2 B + \cos^2 C) = \\&= 3 - (1 - 2 \cos A \cos B \cos C) = \\&= 2 + 2 \cos A \cos B \cos C.\end{aligned}$$

(Szabó István, Debreczen.)

A feladatot még megoldották: Erdős A., Freibauer E., Friedmann B., Goldziher K. és Kármán T., Hrivzák A., Kertész L., Petrogalli G., Spitzer Ö., Szabó K.