

Számítsuk ki $\cos^2 \gamma$ -t. $\gamma = \pi - (\alpha + \beta)$ és így

$$\begin{aligned}\sin \gamma &= \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \text{ és } \cos \gamma = -\cos(\alpha + \beta) = \\ &= -(\cos \alpha \cos \beta - \sin \alpha \sin \beta) \text{ felhasználásával: } \cos^2 \gamma = 1 - \sin^2 \gamma = \\ &= 1 - \sin^2 \alpha \cos^2 \beta - 2 \sin \alpha \sin \beta \cos \alpha \cos \beta - \cos^2 \alpha \sin^2 \beta = \\ &= 1 - (1 - \cos^2 \alpha) \cos^2 \beta - 2 \sin \alpha \sin \beta \cos \alpha \cos \beta - \cos^2 \alpha (1 - \cos^2 \beta) = \\ &= 1 - \cos^2 \alpha - \cos^2 \beta + 2 \cos \alpha \cos \beta (\cos \alpha \cos \beta - \sin \alpha \sin \beta) = \\ &= 1 - \cos^2 \alpha - \cos^2 \beta - 2 \cos \alpha \cos \beta \cos \gamma,\end{aligned}$$

azaz:

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + 2 \cos \alpha \cos \beta \cos \gamma = 1.$$

Neszmélyi András (Pannonhalma, Bencés gimn. VI.)