In memory of György Geréb, psychologist and college professor

1. A trapezium has both an inscribed circle and a circumscribed circle. One of the parallel sides is 10 units long and the radius of the incircle is $\varrho = 5\sqrt{3}$ units. What percentage of the area of the trapezium is the area of the incircle?

2. Two cities are 560 km apart. A car takes 1 hour less to cover this distance than another car because its speed is $10\ \dfrac{\mathrm{km}}{\mathrm{h}}$ greater than that of the other one. Find the speeds of the two cars.

3. The points $P(b,-2)$ and $Q(16b,6)$ lie on the graph of the logarithm function $f\colon x \mapsto \log_a x$, $x \in \mathbb{R}^+$ $(a>0$, $a \neq 1)$. Determine the values of a and b and the equation of the line PQ.

4. Find the domain and range of the expression

$$\frac{\sin 2x + 2\sin x}{\sin 2x - 2\sin x} \cdot \tan^2 \frac{x}{2}.$$

5. Find the equation of the circle of radius $\sqrt{8}$ passing through the point $P(0,2)$ and touched from the outside by the circle of equation

$$x^2 + y^2 + 2x + 6y + 8 = 0.$$

6. Solve the following inequality on the set of real numbers:

$$2\sqrt{\log_3 x} + \sqrt{\log_x 3} \geq 3.$$

7. A convex hexagon with three sides of length $2a$ and three sides of length $5a$ is inscribed in a circle of radius $\sqrt{13}$. Find the area of the hexagon.

8. The first term of a sequence is $a_1 = 1$, and

$$a_{n+1} = \left(1 - \frac{1}{(n+2)^2}\right) \cdot a_n$$

for $n \geq 1$. Express (in closed form) the nth term of the sequence and the product of the first n terms.