## In memory of György Geréb, psychologist and college professor

1. A trapezium has both an inscribed circle and a circumscribed circle. One of the parallel sides is 10 units long and the radius of the incircle is $\varrho=5 \sqrt{3}$ units. What percentage of the area of the trapezium is the area of the incircle?
2. Two cities are 560 km apart. A car takes 1 hour less to cover this distance than another car because its speed is $10 \frac{\mathrm{~km}}{\mathrm{~h}}$ greater than that of the other one. Find the speeds of the two cars.
3. The points $P(b,-2)$ and $Q(16 b, 6)$ lie on the graph of the logarithm function $f: x \mapsto \log _{a} x, x \in \mathbb{R}^{+}(a>0$, $a \neq 1$ ). Determine the values of $a$ and $b$ and the equation of the line $P Q$.
4. Find the domain and range of the expression

$$
\frac{\sin 2 x+2 \sin x}{\sin 2 x-2 \sin x} \cdot \tan ^{2} \frac{x}{2}
$$

5. Find the equation of the circle of radius $\sqrt{8}$ passing through the point $P(0,2)$ and touched from the outside by the circle of equation

$$
x^{2}+y^{2}+2 x+6 y+8=0 .
$$

6. Solve the following inequality on the set of real numbers:

$$
2 \sqrt{\log _{3} x}+\sqrt{\log _{x} 3} \geq 3
$$

7. A convex hexagon with three sides of length $2 a$ and three sides of length $5 a$ is inscribed in a circle of radius $\sqrt{13}$. Find the area of the hexagon.
8. The first term of a sequence is $a_{1}=1$, and

$$
a_{n+1}=\left(1-\frac{1}{(n+2)^{2}}\right) \cdot a_{n}
$$

for $n \geq 1$. Express (in closed form) the $n$th term of the sequence and the product of the first $n$ terms.

