1. Solve the following equation on the set of real numbers:

$$
\frac{2 x+2}{7}=\frac{\left(x^{2}-x-6\right)(x+1)}{x^{2}+2 x-3}
$$

2. For what positive integers $a$ is the value of the following expression also an integer?

$$
\left(\frac{a+1}{1-a}+\frac{a-1}{a+1}-\frac{4 a^{2}}{a^{2}-1}\right):\left(\frac{2}{a^{3}+a^{2}}-\frac{2-2 a+2 a^{2}}{a^{2}}\right)
$$

3. Given that the second coordinates of the points $A(1, a), B(3, b), C(4, c)$ are

$$
a=-\frac{\sin 39^{\circ}+\sin 13^{\circ}}{\sin 26^{\circ} \cdot \cos 13^{\circ}}, \quad b=\sqrt{10^{2+\log _{10} 25}}, \quad c=\left(\frac{1}{\sqrt{5}-2}\right)^{3}-\left(\frac{1}{\sqrt{5}+2}\right)^{3}
$$

determine whether the three points are collinear.
4. What is more favourable: I. If the bank pays $20 \%$ annual interest, and the inflation rate is $15 \%$ per year, or II. if the bank pays $12 \%$ annual interest, and the inflation rate is $7 \%$ per year?
5. The first four terms of an arithmetic progression of integers are $a_{1}, a_{2}, a_{3}, a_{4}$. Show that $1 \cdot a_{1}^{2}+2 \cdot a_{2}^{2}+3 \cdot a_{3}^{2}+4 \cdot a_{4}^{2}$ can be expressed as the sum of two perfect squares.
6. In an acute triangle $A B C$, the circle of diameter $A C$ intersects the line of the altitude from $B$ at the points $D$ and $E$, and the circle of diameter $A B$ intersects the line of the altitude from $C$ at the points $F$ and $G$. Show that the points $D, E, F, G$ lie on a circle.
7. The base of a right pyramid is a triangle $A B C$, the lengths of the sides are $A B=21 \mathrm{~cm}, B C=20 \mathrm{~cm}$ and $C A=13 \mathrm{~cm} . A^{\prime}, B^{\prime}, C^{\prime}$ are points on the corresponding lateral edges, such that $A A^{\prime}=5 \mathrm{~cm}, B B^{\prime}=25 \mathrm{~cm}$ and $C C^{\prime}=4 \mathrm{~cm}$. Find the angle of the planes of triangle $A^{\prime} B^{\prime} C^{\prime}$ and triangle $A B C$.
8. Let $f(x)=2 x^{6}-3 x^{4}+x^{2}$. Prove that $f(\sin \alpha)+f(\cos \alpha)=0$.

