1. Solve the following equation on the set of real numbers:

$$\frac{2x+2}{7} = \frac{(x^2 - x - 6)(x+1)}{x^2 + 2x - 3}$$

2. For what positive integers a is the value of the following expression also an integer?

$$\left(\frac{a+1}{1-a} + \frac{a-1}{a+1} - \frac{4a^2}{a^2-1}\right) : \left(\frac{2}{a^3+a^2} - \frac{2-2a+2a^2}{a^2}\right)$$

**3.** Given that the second coordinates of the points A(1, a), B(3, b), C(4, c) are

$$a = -\frac{\sin 39^\circ + \sin 13^\circ}{\sin 26^\circ \cdot \cos 13^\circ}, \qquad b = \sqrt{10^{2 + \log_{10} 25}}, \qquad c = \left(\frac{1}{\sqrt{5} - 2}\right)^3 - \left(\frac{1}{\sqrt{5} + 2}\right)^3$$

determine whether the three points are collinear.

4. What is more favourable: I. If the bank pays 20% annual interest, and the inflation rate is 15% per year, or II. if the bank pays 12% annual interest, and the inflation rate is 7% per year?

5. The first four terms of an arithmetic progression of integers are  $a_1, a_2, a_3, a_4$ . Show that  $1 \cdot a_1^2 + 2 \cdot a_2^2 + 3 \cdot a_3^2 + 4 \cdot a_4^2$  can be expressed as the sum of two perfect squares.

6. In an acute triangle ABC, the circle of diameter AC intersects the line of the altitude from B at the points D and E, and the circle of diameter AB intersects the line of the altitude from C at the points F and G. Show that the points D, E, F, G lie on a circle.

7. The base of a right pyramid is a triangle ABC, the lengths of the sides are AB = 21 cm, BC = 20 cm and CA = 13 cm. A', B', C' are points on the corresponding lateral edges, such that AA' = 5 cm, BB' = 25 cm and CC' = 4 cm. Find the angle of the planes of triangle A'B'C' and triangle ABC.

8. Let  $f(x) = 2x^6 - 3x^4 + x^2$ . Prove that  $f(\sin \alpha) + f(\cos \alpha) = 0$ .