1. Solve the following equation on the set of real numbers:

$$\frac{2x+2}{7}=\frac{(x^2-x-6)(x+1)}{x^2+2x-3}.$$

2. For what positive integers a is the value of the following expression also an integer?

$$\left(\frac{a+1}{1-a}+\frac{a-1}{a+1}-\frac{4a^2}{a^2-1}\right):\left(\frac{2}{a^3+a^2}-\frac{2-2a+2a^2}{a^2}\right)$$

3. Given that the second coordinates of the points $A(1,a)$, $B(3,b)$, $C(4,c)$ are

$$a=-\frac{\sin 39^\circ+\sin 13^\circ}{\sin 26^\circ\cdot\cos 13^\circ},\qquad b=\sqrt{10^{2+\log_{10}25}},\qquad c=\left(\frac{1}{\sqrt{5}-2}\right)^3-\left(\frac{1}{\sqrt{5}+2}\right)^3$$

determine whether the three points are collinear.

4. What is more favourable: I. If the bank pays 20% annual interest, and the inflation rate is 15% per year, or II. if the bank pays 12% annual interest, and the inflation rate is 7% per year?

5. The first four terms of an arithmetic progression of integers are a_1, a_2, a_3, a_4. Show that $1\cdot a_1^2+2\cdot a_2^2+3\cdot a_3^2+4\cdot a_4^2$ can be expressed as the sum of two perfect squares.

6. In an acute triangle ABC, the circle of diameter AC intersects the line of the altitude from B at the points D and E, and the circle of diameter AB intersects the line of the altitude from C at the points F and G. Show that the points D, E, F, G lie on a circle.

7. The base of a right pyramid is a triangle ABC, the lengths of the sides are $AB=21$ cm, $BC=20$ cm and $CA=13$ cm. A', B', C' are points on the corresponding lateral edges, such that $AA'=5$ cm, $BB'=25$ cm and $CC'=4$ cm. Find the angle of the planes of triangle $A'B'C'$ and triangle ABC.

8. Let $f(x)=2x^6-3x^4+x^2$. Prove that $f(\sin\alpha)+f(\cos\alpha)=0$.