1. $\triangle ABC$ is a given triangle. P_1 is a point inside $\triangle ABC$.

1.Prove that the lines obtained by reflecting P_1A, P_1B, P_1C through the angle bisectors of $\sphericalangle A$, $\sphericalangle B$, $\sphericalangle C$, respectively, meet at a common point P_2.

2.Let A_1, B_1, C_1 be the feet of the perpendiculars from P_1 onto BC, CA and AB, respectively. Let A_2, B_2, C_2 be the feet of the perpendiculars from P_2 onto BC, CA and AB, respectively. Prove that these six points A_1, B_1, C_1, A_2, B_2, C_2 lie on a circle.

3. Prove that the circle in part (b) touches the nine point circle (*Feuerbach's circle*) of $\triangle ABC$ if and only if P_1, P_2 and the center of the circumcircle of $\triangle ABC$ are collinear.

2. An ant is walking inside the region bounded by the curve whose equation is $x^2 + y^2 + xy = 6$. Its path is formed by straight segments parallel to the coordinate axes. It starts at an arbitrary point on the curve and takes off inside the region. When reaching the boundary, it turns by 90° and continues its walk inside the region. When arriving at a point on the boundary which it has already visited, or where it cannot continue its walk according to the given rule, the ant stops. Prove that, sooner or later, and regardless of the starting point, the ant will stop.

3.1. In the plane, we are given the circle C (without its center) and the point P. Is it possible to construct, with a ruler only, the line through P and the center of the circle?

2. In the plane, we are given two circles C_1 and C_2 (without their centers). Construct, with a ruler only, the line through their centers when:

i.) the two circles intersect.

ii.) the two circles touch each other, and their point of contact, T, is marked.

($*$) *iii.*) *We do not know how to solve this problem if the two circles have no common point. It might even be the case that this construction cannot be done at all with a ruler only. Can you do better?*

Note: Part *(iii)* does not belong to the official team contest. However, any progress will be appreciated.