We have toured all the fields of an $8 \times 8$ chessboard with a king. We visited each field exactly once, and the last move took the king back to the field it started from. (The king is allowed to move in the usual way.) By connecting the centres of the fields visited in succession, we obtained a closed polygonal line that does not cut through itself. Prove that the king made at least 28 horizontal and vertical moves altogether.

A competition problem from the Soviet Union

