

Keressük x -et $n^{99} + \frac{1}{n}$ alakban, ahol n 1-nél nagyobb egész. Ekkor

$$\begin{aligned} \{x^{2k+1}\} &= \left\{ \left(n^{99} + \frac{1}{n} \right)^{2k+1} \right\} = \left\{ (n^{99})^{2k+1} + \binom{2k+1}{1} (n^{99})^{2k} \cdot \frac{1}{n} + \right. \\ &\quad \left. + \binom{2k+1}{2} (n^{99})^{2k-1} \cdot \frac{1}{n^2} + \dots + \binom{2k+1}{2k} n^{99} \cdot \frac{1}{n^{2k}} + \frac{1}{n^{2k+1}} \right\} = \\ &= \left\{ \sum_{i=0}^{2k+1} \binom{2k+1}{i} n^{198k-100i+99} \right\}. \end{aligned}$$

Ha $0 \leq k \leq 49$ és $0 \leq i \leq 2k$, akkor $198k - 100i + 99 > 0$ egész. Tehát ebben az esetben

$$\{x^{2k+1}\} = \left\{ \binom{2k+1}{2k+1} n^{198k-100(2k+1)+99} \right\} = \left\{ \left(\frac{1}{n} \right)^{2k+1} \right\} = \left(\frac{1}{n} \right)^{2k+1}.$$

Így

$$\begin{aligned} \{x\} + \{x^3\} + \dots + \{x^{99}\} &= \frac{1}{n} + \frac{1}{n^3} + \dots + \frac{1}{n^{99}} = \\ &= \frac{1}{n} \cdot \frac{1 - \left(\frac{1}{n^2}\right)^{50}}{1 - \frac{1}{n^2}} < \frac{1}{n} \cdot \frac{1}{1 - \frac{1}{n^2}} = \frac{n}{n^2 - 1} < \frac{n+1}{n^2 - 1} = \frac{1}{n-1}. \end{aligned}$$

Azaz, ha $n > 2^{99} + 1$, úgy $x = n^{99} + \frac{1}{n}$ választása mellett

$$\{x\} + \{x^3\} + \dots + \{x^{99}\} < \frac{1}{n-1} \leq \frac{1}{2^{99}}.$$

Sarlós Ferenc (Baja, III. Béla Gimn., 12. évf.) dolgozata alapján