

1°. Ismeretes, hogy (*Mathematikai Gyakorlókönyv II. 360. f.*)

$$(1) \quad \operatorname{tg} \alpha + \operatorname{tg} \beta + \operatorname{tg} \gamma = \operatorname{tg} \alpha \operatorname{tg} \beta \operatorname{tg} \gamma.$$

Ha eme egyenlet mindkét oldalát négyzetre emeljük, ered

$$\begin{aligned} A &= \operatorname{tg}^2 \alpha \operatorname{tg}^2 \beta \operatorname{tg}^2 \gamma - (\operatorname{tg}^2 \alpha + \operatorname{tg}^2 \beta + \operatorname{tg}^2 \gamma) = \\ &= 2(\operatorname{tg} \alpha \operatorname{tg} \beta + \operatorname{tg} \alpha \operatorname{tg} \gamma + \operatorname{tg} \beta \operatorname{tg} \gamma) = \\ &= 2 \sec \alpha \sec \beta \sec \gamma (\sin \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma \cos \beta + \sin \alpha \sin \gamma \cos \alpha) = \\ &= 2 \sec \alpha \sec \beta \sec \gamma (\sin \alpha \sin \beta \cos \gamma + \sin^2 \gamma) = \\ &= 2 \sec \alpha \sec \beta \sec \gamma [1 + \cos \gamma (\sin \alpha \sin \beta - \cos \gamma)] = \\ &= 2 \sec \alpha \sec \beta \sec \gamma [1 + \cos \alpha \cos \beta \cos \gamma] = 2 + 2 \sec \alpha \sec \beta \sec \gamma. \end{aligned}$$

2°. Ha (1)-et köbre emeljük, ered

$$\begin{aligned} B &= \operatorname{tg}^3 \alpha \operatorname{tg}^3 \beta \operatorname{tg}^3 \gamma - (\operatorname{tg}^3 \alpha + \operatorname{tg}^3 \beta + \operatorname{tg}^3 \gamma) = \\ &= 3[\operatorname{tg}^2 \alpha \operatorname{tg} \beta + \operatorname{tg} \alpha \operatorname{tg}^2 \beta + \operatorname{tg}^2 \alpha \operatorname{tg} \gamma + \operatorname{tg} \alpha \operatorname{tg}^2 \gamma + \operatorname{tg}^2 \beta \operatorname{tg} \gamma + \operatorname{tg} \beta \operatorname{tg}^2 \gamma + \\ &\quad + 3\operatorname{tg} \alpha \operatorname{tg} \beta \operatorname{tg} \gamma - \operatorname{tg} \alpha \operatorname{tg} \beta \operatorname{tg} \gamma] = \\ &= 3[\operatorname{tg} \alpha \operatorname{tg} \beta (\operatorname{tg} \alpha + \operatorname{tg} \beta + \operatorname{tg} \gamma) + \operatorname{tg} \alpha \operatorname{tg} \gamma (\operatorname{tg} \alpha + \operatorname{tg} \beta + \operatorname{tg} \gamma) + \\ &\quad + \operatorname{tg} \beta \operatorname{tg} \gamma (\operatorname{tg} \alpha + \operatorname{tg} \beta + \operatorname{tg} \gamma) - \operatorname{tg} \alpha \operatorname{tg} \beta \operatorname{tg} \gamma] = \\ &= 3\operatorname{tg} \alpha \operatorname{tg} \beta \operatorname{tg} \gamma [\operatorname{tg} \alpha \operatorname{tg} \beta + \operatorname{tg} \alpha \operatorname{tg} \gamma + \operatorname{tg} \beta \operatorname{tg} \gamma - 1]. \end{aligned}$$

De, mint 1°-ben láttuk, a szögletes zárójelben álló kifejezés:

$$\sec \alpha \sec \beta \sec \gamma + 1,$$

s így

$$B = 3 \operatorname{tg} \alpha \operatorname{tg} \beta \operatorname{tg} \gamma [\sec \alpha \sec \beta \sec \gamma + 1 - 1] = \frac{3 \operatorname{tg} \alpha \operatorname{tg} \beta \operatorname{tg} \gamma}{\cos \alpha \cos \beta \cos \gamma}.$$

(Kiss Ernő. Budapest.)