

1°. A $\sin^3 \alpha = 3 \sin \alpha - 4 \sin^3 \alpha$ képlet alapján:

$$\begin{aligned}
 \sin^3 \alpha + \sin^3 \beta + \sin^3 \gamma &= \frac{1}{4} [3(\sin \alpha + \sin \beta + \sin \gamma) + \\
 &\quad + (-\sin 3\alpha - \sin 3\beta - \sin 3\gamma)] = \\
 &= \frac{1}{4} \{3[2 \sin \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta) + 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}\gamma] + \\
 &\quad + [-2 \sin \frac{3}{2}(\alpha + \beta) \cos \frac{3}{2}(\alpha - \beta) - 2 \sin \frac{3}{2}\gamma \cos \frac{3}{2}\gamma]\} = \\
 &= \frac{1}{4} [6 \cos \frac{\gamma}{2} (\cos \frac{1}{2}(\alpha - \beta) + \cos \frac{1}{2}(\alpha + \beta)) + \\
 &\quad + 2 \cos \frac{3}{2}\gamma (\cos \frac{3}{2}(\alpha - \beta) + (\cos \frac{3}{2}(\alpha + \beta)))] = \\
 &= \frac{1}{4} \left[12 \cos \frac{\alpha}{2} \cos \frac{\beta}{2} \cos \frac{\gamma}{2} + 4 \cos \frac{3}{2}\alpha \cos \frac{3}{2}\beta \cos \frac{3}{2}\gamma \right] = \\
 &= 3 \cos \frac{\alpha}{2} \cos \frac{\beta}{2} \cos \frac{\gamma}{2} + \cos \frac{3\alpha}{2} \cos \frac{3\beta}{2} \cos \frac{3\gamma}{2}.
 \end{aligned}$$

2°. A $\cos^3 \alpha = 4 \cos^3 \alpha - 3 \cos \alpha$ képlet alapján:

$$\begin{aligned}
 \cos^3 \alpha + \cos^3 \beta + \cos^3 \gamma &= \frac{1}{4} [3(\cos \alpha + \cos \beta + \cos \gamma) + \\
 &\quad + (\cos 3\alpha + \cos 3\beta + \cos 3\gamma)] = \\
 &= \frac{1}{4} \{3[2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta) - 2 \sin^2 \frac{\gamma}{2} + 1] + \\
 &\quad + 2 \cos \frac{3}{2}(\alpha + \beta) \cos \frac{3}{2}(\alpha - \beta) - 2 \sin^2 \frac{3\gamma}{2} + 1\} = \\
 &= \frac{1}{4} \{6 \sin \frac{\gamma}{2} [\cos \frac{1}{2}(\alpha - \beta) - \cos \frac{1}{2}(\alpha + \beta)] - \\
 &\quad - 2 \sin \frac{3\gamma}{2} [\cos \frac{3}{2}(\alpha - \beta) - \cos \frac{3}{2}(\alpha + \beta)] + 4\} = \\
 &= \frac{1}{4} \left[12 \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2} - 4 \sin \frac{3\alpha}{2} \sin \frac{3\beta}{2} \sin \frac{3\gamma}{2} + 4 \right] = \\
 &= 3 \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2} - \sin \frac{3\alpha}{2} \sin \frac{3\beta}{2} \sin \frac{3\gamma}{2} + 1.
 \end{aligned}$$

(Sárközy Pál, Pannonhalma.)