

(1) Minthogy

$$\operatorname{ctg}(\beta + \gamma) = \frac{\operatorname{ctg}\beta \operatorname{ctg}\gamma - 1}{\operatorname{ctg}\beta + \operatorname{ctg}\gamma}$$

és

$$\operatorname{ctg}(\beta + \gamma) = -\operatorname{ctg}\alpha,$$

azért

$$\operatorname{ctg}\beta \operatorname{ctg}\gamma = 1 - \operatorname{ctg}\alpha(\operatorname{ctg}\beta + \operatorname{ctg}\gamma).$$

Így tehát

$$\begin{aligned} \operatorname{ctg}\alpha \operatorname{ctg}\beta + \operatorname{ctg}\beta \operatorname{ctg}\gamma + \operatorname{ctg}\gamma \operatorname{ctg}\alpha &= \operatorname{ctg}\alpha(\operatorname{ctg}\beta + \operatorname{ctg}\gamma) + \\ + \operatorname{ctg}\beta \operatorname{ctg}\gamma &= \operatorname{ctg}\alpha(\operatorname{ctg}\beta + \operatorname{ctg}\gamma) + 1 - \operatorname{ctg}\alpha(\operatorname{ctg}\beta + \operatorname{ctg}\gamma) = 1. \end{aligned}$$

(2) Minthogy

$$\operatorname{tg} \frac{\beta + \gamma}{2} = \operatorname{tg} \left(90^\circ - \frac{\alpha}{2} \right) = \operatorname{ctg} \frac{\alpha}{2} = \frac{\operatorname{tg} \frac{\beta}{2} + \operatorname{tg} \frac{\gamma}{2}}{1 - \operatorname{tg} \frac{\beta}{2} \operatorname{tg} \frac{\gamma}{2}},$$

azért

$$\operatorname{tg} \frac{\beta}{2} + \operatorname{tg} \frac{\gamma}{2} = \operatorname{ctg} \frac{\alpha}{2} - \operatorname{ctg} \frac{\alpha}{2} \operatorname{tg} \frac{\beta}{2} \operatorname{tg} \frac{\gamma}{2}.$$

Így tehát

$$\begin{aligned} \operatorname{tg} \frac{\alpha}{2} \operatorname{tg} \frac{\beta}{2} + \operatorname{tg} \frac{\beta}{2} \operatorname{tg} \frac{\gamma}{2} + \operatorname{tg} \frac{\gamma}{2} \operatorname{tg} \frac{\alpha}{2} &= \operatorname{tg} \frac{\alpha}{2} = \operatorname{tg} \frac{\alpha}{2} \left(\operatorname{tg} \frac{\beta}{2} + \operatorname{tg} \frac{\alpha}{2} \right) + \\ + \operatorname{tg} \frac{\beta}{2} \operatorname{tg} \frac{\gamma}{2} &= \operatorname{tg} \frac{\alpha}{2} \operatorname{ctg} \frac{\alpha}{2} - \operatorname{tg} \frac{\alpha}{2} \operatorname{ctg} \frac{\alpha}{2} - \operatorname{tg} \frac{\alpha}{2} \operatorname{ctg} \frac{\alpha}{2} \operatorname{tg} \frac{\beta}{2} \operatorname{tg} \frac{\gamma}{2} + \operatorname{tg} \frac{\beta}{2} \operatorname{tg} \frac{\gamma}{2} = \\ &= 1 - \operatorname{tg} \frac{\beta}{2} \operatorname{tg} \frac{\gamma}{2} + \operatorname{tg} \frac{\beta}{2} \operatorname{tg} \frac{\gamma}{2} = 1. \end{aligned}$$

(Krausz Jenő, Székes-Fehérvár.)

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