I.

$$\begin{gathered}
\cos^2 A+\cos^2 B+\cos^2 C= \\
=\cos A\cos[180^\circ-(B+C)]+\cos B\cos[180^\circ-(A+C)]+\cos^2 C= \\
=-\cos A\cos(B+C)-\cos B\cos(A+C)+\cos^2 C= \\
=-2\cos A\cos B\cos C+\sin A\sin B\sin C+\cos B\sin A\sin C+\cos^2 C= \\
=\sin C(\sin A\cos B+\cos A\sin B)+\cos^2 C-2\cos A\cos B\cos C= \\
=\sin C\sin(A+B)+\cos^2 C-2\cos A\cos B\cos C= \\
=\sin^2 C+\cos^2 C-2\cos A\cos B\cos C= \\
=1-2\cos A\cos B\cos C.
\end{gathered}$$

II.

$$\begin{gathered}
\sin^2 A+\sin^2 B+\sin^2 C= \\
=1-\cos^2 A+1-\cos^2 B+1-\cos^2 C= \\
=3-(\cos^2 A+\cos^2 B+\cos^2 C)= \\
=3-(1-2\cos A\cos B\cos C)= \\
=2+2\cos A\cos B\cos C.
\end{gathered}$$

(Szabó István, Debreczen.)

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