Paradoxes challenge our logical abilities as they show in a striking way how easy it is to come to a contradictory conclusion through apparently correct reasoning. Only when scrutinizing a paradox do we realize how important it is to check the result after solving a problem, to see whether we have committed a logical error.

Firstly, consider the paradox of unexpected execution. The judge sentences the accused to death, and orders that the execution has to be carried out unexpectedly on some day next week. The advocate consoles the convict:

– There's no need to worry. As executions are done only on weekdays, if you are still alive on Thursday evening, you can't be executed on Friday, since this wouldn't be unexpected for you. So Friday is out of the question. Consequently, you have to be executed not later than Thursday. However, if you are still alive on Wednesday night, you can't be executed on Thursday, since this wouldn't be unexpected for you, either. Reasoning similarly, Wednesday, Tuesday and also Monday are out of the question.

The convict returns to his cell relieved, but then the door of his cell opens up, say, on Wednesday ...

Where is the fallacy in the advocate's argument?

What did the judge say? „The execution has to be done unexpectedly next week." These two conditions are inconsistent; if it is known to happen sometime next week, then it is not unexpected. If I know for example that I am going to get a letter from the tax office next week, then the letter is not going to be unexpected, no matter which day I receive it, and I am going to check the mail each day, because I am expecting it. It would come unexpectedly if I did not count on it at all.

All the convict has to do is get up early every morning, get dressed, and announce when the door opens: – I was expecting you, sirs.

The advocate's argument suggests that being unexpected means that the day of the execution is not known. As we have seen, this does not make the day unexpected.

Another version of the paradox is when the commander announces that an unexpected drill will take place the following week. He obviously wants to know how quickly the soldiers react to the alarm if they do not expect it. His announcement is counterproductive since the soldiers will prepare for it each night.

This paradox was based on the ambiguity of being *unexpected.* The next one is going to be more sophisticated.

Two passengers A and B are travelling on a train. Both of them think of a number. When the ticket inspector enters, both whisper to him the number they thought of. Then the ticket inspector says:

– You have thought of different natural numbers, gentlemen, and you have no way of guessing whose number is greater.

The ticket inspector then leaves the compartment and the passengers start thinking. A for instance, who thought of, say, 23, argues this way: – B can't have thought of 1, otherwise he would know that my number is greater. He also knows that I couldn't have thought of 1, either, for the same reason. So none of us have thought of 1. But then B couldn't have thought of 2, either, otherwise he would know again that my number is greater, and as he argues the same way as I do, he also knows that I couldn't have thought of 2, either. Thus, 2 is also out of the question. Following the same line of reasoning A discards 3 and 4, etc. until he reaches 23, which according to the ticket inspector's statement still could not has been his number. He then concludes that the ticket inspector's statement is false, even if he has to admit that he does not have the vaguest idea whose number is bigger.

Readers are kindly asked to try and find the solution by themselves. The reward is a rare „Aha!"-experience.

For a better understanding let us start with a simpler case. Suppose that neither the passengers nor the ticket inspector know each other. When the ticket inspector enters the compartment, he thinks: – None of them knows that my name is Jones. – Then he also says it loudly:

– You don't know that my name is Jones.

Think about it, when he starts his announcement, it is obviously true. When he finishes it – it is not true any more! Due to the sentence they have learned his name, the sentence turned false, and so the ticket inspector can not say it any more.

Let us return now to the original paradox. Suppose that A thought of 2, and B of 11. As long as they sit silently next to each other, they can not decide whose number is greater. This fact is announced by the ticket inspector. Having done the sentence turnes false immediately, since on hearing this, A can deduce that his number is smaller. (B is certainly still helpless.) The ticket inspector's statement should be interpreted this way: „You haven't been able to decide so far whose number is greater."

And if A has thought of 3? In this case, on hearing the ticket inspector's statement A only knows that B has not thought of 1. But he could have thought of 2! In fact, A does not know whether upon hearing the sentence B *already knows whose number is greater.* A can discard B's 2, only if the ticket inspector says it once again: „You still don't know whose number is greater." And if A thought of 6, for example, then the ticket inspector has to repeat his statement five times, until A figures out that his number is the smaller one.

A and B can decide whose number is greater by questioning each other. If A's number is 5 and B's one is greater, and A is the first to ask („Do you know whose number is greater?"), then the following conversation might take place:

B	A
– No (from this A knows that B has not thought of 1)	– I don't, either (B also knows, that
– Still don't (from this A knows that B has not thought of 2)	– I don't, either (B also knows)
– Not yet (B of not 3)	– I don't, either. (A of not 3)
– Still not yet (B of not 4)	– I do know it now!
– And so do I!	