Problems of the 1986 Kürschák József Competition

1. Prove that three semi lines starting from a given point contain three face diagonals of a cuboid if and only if the semi lines include pairwise acute angles such that their sum is 180°.

2. Let us assume that n is a positive integral number greater than two. Find the maximum value for h and the minimum value for H such that

$$h < \frac{a_1}{a_1 + a_2} + \frac{a_2}{a_2 + a_3} + \ldots + \frac{a_n}{a_n + a_1} < H,$$

holds for any positive numbers a_1, a_2, …, a_n.

3. A and B play the following game. They arbitrarily select from among the first 100 positive integral numbers k ones. If the sum of the selected numbers is even then A wins, if their sum is odd then B is the winner. For what values of k are equal the chances for A and B?