**1.** Circle $k$ and the circumcircle of the triangle $ABC$ are touching externally. Circle $k$ is also touching the rays $AB$ and $AC$ at the points $P$ and $Q$, respectively. Prove that the midpoint of the segment $PQ$ is the centre of the excircle touching the side $BC$ of the triangle $ABC$.

**2.** Find the smallest positive integer different from 2004 with the property that there exists a polynomial $f(x)$ of integer coefficients such that the equation $f(x) = 2004$ has at least one integer solution and the equation $f(x) = n$ has at least 2004 distinct integer solutions.

**3.** Some points are given along the circumference of a circle, each of them is either red or blue. The coloured points are subjects to the following two operations:

($a$) a red point can be inserted anywhere along the circle while the colours of its two neighbours are changed from red to blue and vice versa;

($b$) if there are at least three coloured points present and there is a red one among them then a red point can be removed while its two neighbours are switching colours.

Starting with two blue points is it possible to end up with two red points after an appropriate sequence of the above operations?