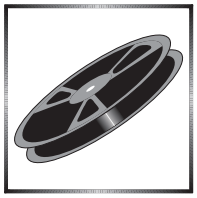

Mathematics is a profitable investment for publishers and producers. The novel, *Le théorème du perroquet* (The Theorem of the Parrot) by the French Denis Guedj has become a hot best seller, and the play *Proof* by the American David Auburn won the 2001 Pulitzer Prize for drama. The waves of media attention following the world-renowned proof by Englishman Andrew Wiles[1] were just starting to calm down when it was reported that Russel Crowe, whose role as gladiator in a typical Hollywood interpretation of Roman history epitomized the eternal American character of the lone avenger, was preparing for a new role. Another lone hero, this time a mathematical genius, was not fighting petty pseudo-Roman tyrants but rather fighting himself, his own creative force and his illness, which was diagnosed as incurable. Starring in the film about the life and mathematics of John Nash, he intended to impersonate again the classic American myth. The success of the film is now well known: it won the Oscar for best film in 2002.

There have never been so many mathematics related movies produced in Hollywood and independent workshops as there have been in the past ten years. A few decades ago, mathematicians were only portrayed as absent-minded professors (the only exception was in *Straw Dogs,* 1971, in which, between solving two equations, a mathematician seeking a secluded place in the country takes a bloody revenge on the gang who have raped his wife.) Since the 90's, a wide spectrum of characters and motives has appeared, from the action heroes fighting dinosaurs (*Jurassic Park*) to complex characters based on real scientists.

No doubt, the history of mathematics cannot boast as many lives full of adventure as physics or astronomy can, where scientists challenge the prevailing paradigm at all times: the stories of Archimedes setting fire to Roman ships or Galileo shouting „And yet it moves" in the face of the inquisitor are more impressive in technicolor than that of Andrew Wiles spending eight years of his life proving Fermat's Last Theorem[2]. Wiles, nevertheless, did become a media star; he was even featured on the front page of The Times. He was not portrayed by Hollywood but in a feature by the BBC, the capital of popular science[3]. That is what the best-selling book of the 90's, Fermat's Last Theorem by the director Simon Singh was based on. It is unlikely that there will be more films and books like these. It is not that there is no mathematical breakthrough to be expected similar to Wiles', but because, as a mathematician put it, Fermat's last theorem is probably the last open problem in mathematics that could be stated in such simple terms that it could even be explained to one's grandmother.

Most achievements in mathematics are hard to communicate, so it's no wonder that the exceptions that find their way onto the movie screen focus on the human drama of the creative intellect instead. The biographical film *A Hill on the Dark Side of the Moon,* 1993 concentrates on the storms in the family of the neurotic Sonia Kovalevskaya, a scientist suffering from an inferiority complex; in *Infinity,* 1996, the Nobel Prize winner Richard Feynman spends more time with his wife dying of tuberculosis than with his integrals; and the crew of *Morte di un matematico napoletano* (Death of a Neapolitain Mathematician), 1992 was only interested in the tumultuous week preceding the suicide of the Italian Renato Cacciopolli. The life of the adolescent genius Galois inspired two movies only a few years apart, but both the French (*Évariste Galois,* 1965) and the Italian version (*Non ho tempo,* 1972) emphasize the political activity of the great mathematician.

A Beautiful Mind seems to follow the same course: the story of the life of John Nash suffering from paranoid schizophrenia contains all it takes to sink hearts and to get the handkerchiefs out. This time, however, mathematics and in particular game theory, the part of the mathematics of John Nash that can be understood by the public[4] receives a much greater role than the annoying clichés used in other movies, like New Age gurus who can make order out of chaos in a glance when looking at a blackboard full of formulas (*Good Will Hunting*).

Though Ron Howard's film only devotes the first third of the story to the mathematics of Nash – roughly proportional to the active part of his life – he illustrates it with much more emphasis and ingenuity than Hollywood usually does.

Mathematical problems, proofs and theoretical reasoning had been parts (or rather small fractions) of the plots of earlier films about scientists: the space scientist of *Contact,* 1977 lectures on prime numbers to glassy-eyed Pentagon officials, *Una pura formalità* (A pure formality), 1994 contains a beautiful and poetic description of how parallels meet at infinity; in the love story of *The Mirror Has Two Faces,* 1996 a mathematics professor explains the conjecture of twin primes to his art-student girlfriend. The most charming example is certainly found in the musical *Merry Andrew,* 1958 where the comedian Danny Kaye sings a cheerful song about the Pythagorean theorem. In A Beautiful Mind,

[1] *Fermat's Last Theorem,* by Simon Singh.

[2] In fact, Wiles proved a much more general statement, the so-called Taniyama conjecture, or rather a part of it. The statement of Fermat's last theorem follows as a corollary. (the editors)

[3] The entertainment industry also has its contribution. In 2001, The New York Theatre Company staged the Musical *Fermat's Last Tango,* with the revealing subtitle: A Musical Fantasy inspired by Andrew Wiles and his encounters with Fermat's Last Theorem.

[4] Nash had landmark achievements in the theory of nonlinear partial differential equations and in differential geometry. He was 21 when he found the theorem for which he was later on awarded the Nobel Prize in Economics. The whole paper is only a little more than a page. (the editors)

however, the whole first third of the plot is organized around the Nash theory: the screenplay is a series of loosely related short presentations (building on one another). The storyline about the start of Nash's career which links them together is more of a pretext for the authors to provide a tasty way of making the unaware public swallow theoretical mathematics, much like a bitter pill disguised. The „Eureka" scene in the pub is a true didactic feat. Nash is among fellow students hanging out with a few girls. Contemplating aloud, he says that if each of them goes after the same blonde then one of them will win that woman, but the rest of the boys will be refused by the disappointed other girls since no one likes being chosen as number two. The optimum strategy suggested is not so easy to understand but the expected gain of all players is quite clear...

The flock of hungry pigeons, the fierce games of Go, the scenes on the „how to pick up girls" problem not only bring the theories closer to the public but also reveal the everyday meaning of game theory. Howard also finds time in the film to mock at the stereotypes of mathematics in popular films: the pretentious scene in the secret defense centre where the genius' ability to see a pattern in the chaos of numbers is revealed will later prove to be a schizophrenic delusion of Nash – it is as if even mathematicians in their mad visions pictured themselves living out Hollywood clichés.

The „Dream Factory" had an easy job with Nash since game theory is the area of mathematics that often inspires dramaturgy. A beautiful example is the so-called chicken dilemma that came to textbooks and terminology from a classic James Dean film. Chicken is a popular game in motorized America. In *Rebel without a Cause,* it appears in its classical form: the two players race neck and neck towards an abyss, and thus it may happen that the winner loses his car (and his life) despite the other player proving his cowardice by stepping on the brakes or steering the vehicle to the side. In later films (see *Footloose* or the ending of *Cry Baby*), the heroes drive towards each other: whoever gets out of the way first, loses the game. The chicken race is a simple example of a symmetric zero-sum game where the players will definitely not cooperate, since their goal is to humiliate the opponent.

From the point of view of game theory, there is no dominant equilibrium in the chicken race, that is, there is no strategy that maximizes the gain independently of the decision of the other player. In spite of that, it seems evident even without a table of strategies that if the main goal is to survive the duel then steering the car aside is true right solution for each player, the sooner the better. However, the motion picture does not follow the mathematical model, nor does reality sometimes, as is shown by the example of the Cuban Missile Crisis, which has had several screen adaptations, too: either because of the stronger dramatic effect or under outside constraints, the players choose to take the risk in order to achieve total victory. *Cry Baby,* just like John Fitzgerald Kennedy does not want to be seen as a chicken, even though there are other people (or the whole humankind) sitting in the car with him. It is remarkable that as soon as screenplay writers entrust warfare to supercomputers, the moral of the story justifies game theory: In the science fiction story of *War Games,* 1987 the computer of the Department of Defense comes to the conclusion that the only right strategy in the chicken race of the cold war of nuclear threat is for both parties to call a halt. In order to do that, it is necessary to be able to move from the given self-centred game scenario to another, more general game. This wise conclusion of the computer did not seem to be shared by the crew of *13 Days,* which is full of heroic pathos.

Another dramaturgical cliché is the prisoner's dilemma, the latest example of which has been *Return to Paradise,* 1998, a sentimental drama shown in multiplex movie theatres. The situation is hard to evaluate from the point of view of pure game theory. Three young Americans go on holiday to Malaysia, then two of them return to the States. They are soon informed that their friend has been imprisoned by the local authorities for the drug use of all three of them. Being the only one responsible for possessing a commercial quantity of drugs, he is going to be sentenced to death. If either of his two friends returns and testifies for him (proving that the three of them were only consumers), he will get six years in prison. If they both testify, the penalty is 3 years for each of them, but if neither of them is willing to sacrifice himself, their friend will pay with his life instead of going to prison.

Mathematically, the problem is simple if we are able to answer the really difficult question: the evaluation of the alternatives. This is not determined by mathematics but according to the preferences of the characters and the authors. If the life of their friend is what they value most, the solution of the chicken race is that they both „get out of the car", and testify. If their personal freedom is more important, there will be a trivial dominant equilibrium: independent of the decision of the other, each player is better off staying at home, far from Malaysia. No game theory is needed to see that. We could make up the appropriate normal form for each situation, but the only surprising result would be (and this is indeed shocking) that the same mathematics leads to quite a different conclusion in each case.

The neutrality of the mathematical apparatus reveals the enormous risk of technocratic reasoning: the presence of mathematics gives a false sensation of objectivity and exactness in an analysis that basically depends on preferences.

The trivial solutions were obviously avoided by the professional dramaturgy of the film. The mathematical model itself is most interesting if there is a conflict of values: it provides a better model of reality if the negative value assigned to the „utility" of having one's friend executed is compared to the expected number of years in prison, that is set between -6 and -3. The normal form will then be as follows:

	A returns	**A stays at home**
B returns	B: -3, A: -3	B: -6, A: 0
B stays at home	B: 0, A: -6	B: $-3/-6$, A: $-3/-6$

Should we choose either the version equivalent to three years in prison or the other extreme value of six years, there will be a dominant equilibrium again: independent of the decision of the other one, each player will gain more by

staying at home. The authors of *Return to Paradise,* however, chose the most dramatic case: it is impossible to make a movie out of the two heroes both preferring freedom to principle. Next to selecting the most uncertain alternative from the point of view of game theory, the authors also „cheated" by changing the rules as the story developed: one hero did testify, but the friend was still sentenced to death by the ruthless judges, and executed against the orange sunrise, in front of the eyes of the hero sobbing behind the bars.

As shown by the examples, the „Dream Factory" has not only found mathematical topics recently, but also used some results of the theory. A Beautiful Mind not only goes beyond a „psychopathological epic" and offers a mental adventure through the game theory based analysis of trivial situations, but also provides a fine analogy of a harmonic relationship between mathematics and mainstream films. His own hallucinations significantly motivate Nash in his research, and it is only beyond a certain critical point that they become a hindrance, and destroy his personality. The two imaginary characters, the helpful college fellow and the threatening government agent stand on different sides of this borderline. The combination of pictures and the purest mathematics may be of mutual advantage as soon as film authors are willing to do away with clichés and recognize the creative opportunities provided by mathematics: what is good for mathematics may also be good for Hollywood, and even the converse of this statement may contain some truth. Or, as Nash himself indicated when receiving the Nobel Prize: „It's a shame that the majority of academia fails to understand that the essence of mathematics is itself an art." This sentence could well be addressed to the American Film Academy, too.[5]

[5]With many thanks to *János Pataki* who helped my work with useful advice.