

$$\gamma = 180^\circ - (\alpha + \beta) \quad \cos \gamma = -\cos(\alpha + \beta)$$

értéket beírva, kapjuk a baloldaltól, hogy

$$\begin{aligned} & \cos^2 \alpha + \cos^2 \beta + \cos^2(\alpha + \beta) - 2 \cos \alpha \cos \beta \cos(\alpha + \beta) = \\ & = \cos^2 \alpha + \cos^2 \beta + (\cos \alpha \cdot \cos \beta - \sin \alpha \sin \beta)^2 - \\ & \quad - 2 \cos \alpha \cos \beta (\cos \alpha \cos \beta - \sin \alpha \sin \beta) = \\ & = \cos^2 \alpha + \cos^2 \beta + \cos^2 \alpha \cdot \cos^2 \beta + \sin^2 \alpha \cdot \sin^2 \beta - 2 \cos^2 \alpha \cos^2 \beta = \\ & = \cos^2 \alpha + \cos^2 \beta - \cos^2 \alpha \cdot \cos^2 \beta + \sin^2 \alpha \cdot \sin^2 \beta = \\ & = \cos^2 \alpha + \cos^2 \beta - \cos^2 \alpha \cdot \cos^2 \beta + (1 - \cos^2 \alpha) \cdot (1 - \cos^2 \beta) = \\ & = \cos^2 \alpha + \cos^2 \beta - \cos^2 \alpha \cdot \cos^2 \beta + 1 - \cos^2 \alpha - \cos^2 \beta + \cos^2 \alpha \cdot \cos^2 \beta = 1. \end{aligned}$$

Megoldotta: Barabás Gy., Durst E., Fülöp J., Kántor S., Révész P., Villányi O.