

I. megoldás: Ismeretes, hogy $a = 2r \cdot \sin \alpha$, $b = 2r \cdot \sin \beta$, $c = 2r \cdot \sin \gamma$. A háromszög területét jelöljük t -vel.

$$\begin{aligned} 2r(p_a \sin \alpha + p_b \cdot \sin \beta + p_c \cdot \sin \gamma) &= p_a \cdot a + p_b \cdot b + p_c \cdot c = 2t = \\ &= ab \cdot \sin \gamma = 2r \sin \alpha \cdot 2r \sin \beta \cdot \sin \gamma. \end{aligned}$$

Mindkét oldalt $2r$ -rel osztva:

$$p_a \sin \alpha + p_b \cdot \sin \beta + p_c \cdot \sin \gamma = 2r \sin \alpha \sin \beta \sin \gamma.$$

Tar Katalin (Keszthely, premontrei gimn. V.)

II. megoldás: $p_a = r \cdot \cos \alpha$, $p_b = r \cdot \cos \beta$, $p_c = r \cdot \cos \gamma$.

$$\begin{aligned} p_a \sin \alpha + p_b \sin \beta + p_c \cos \gamma &= r(\sin \alpha \cos \alpha + \sin \beta \cos \beta + \sin \gamma \cos \gamma) = \\ &= \frac{r}{2}(\sin 2\alpha + \sin 2\beta + 2 \sin \gamma \cos \gamma) = \frac{r}{2}[2 \sin(\alpha + \beta) \cos(\alpha - \beta) + \\ &\quad + 2 \sin \gamma \cos \gamma] = \frac{r}{2}[2 \sin \gamma \cos(\alpha - \beta) + 2 \sin \gamma \cos \gamma] = \\ &= r \sin \gamma [\cos(\alpha - \beta) - \cos(\alpha + \beta)] = r \sin \gamma (\cos \alpha \cos \beta + \\ &\quad + \sin \alpha \sin \beta - \cos \alpha \cos \beta + \sin \alpha \sin \beta) = 2r \sin \alpha \sin \beta \sin \gamma. \end{aligned}$$

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